Teacher's Guide
Level 3 – Book 3

SRA
MATHEMATICS
LEARNING SYSTEM TEXT

Canadian Publication

MATHEMATICS LEARNING SYSTEM

The Canadian publication of *Mathematics Learning System* is completely metric.

On some pages, major revisions have been made. In several cases the goal of the page has been altered, as indicated below:

Page 185 Exploring centimetres as units of measure, and practice in measuring line segments
Page 186 Introducing millimetres as units of measure
Page 187 Introducing the metre and the metrestick, and measuring in centimetres with a metrestick
Page 188 Practice in measuring with a metrestick
Page 189 Measuring with a metretape or a metre string
Page 190 Practice in selecting an appropriate unit of measure

On the following pages, the answers are as given below:

Page 181

km	cm	m
cm	cm	m

Page 185

2a. 13 cm
b. 11 cm
c. 5 cm
d. 2 cm
e 8 cm
f. 15 cm
g. 12 cm
h. 3 cm
i. 10 cm
j. 6 cm
k. 8 cm
l. 3 cm

Page 186

a.	3	b.	20
	4		30
	6		80
	9		100
	10		60
			70

Page 188

4a.	2	b.	400
	3		600
	4		800
	6		1000
	8		300
	10		200

Page 189

Answers will vary

Page 190

Answers are as given in this Teacher's Guide for page 191.

Page 191

1. 116 cm
2. 135 cm
3. 196 cm
4. 250 cm
5. 227 cm
6. 595 cm
7. 110 cm
8. 200 cm
9. 105 cm
10. 302 cm

Page 191 Practice with related measurements (metres and centimetres)
Page 192 Introduction to kilometres as units of measure for long distances

Substantial revisions have been made in the pupils' textbook for Canadian use, on pages 181, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, and 196, and minor ones on other pages. These include several new illustrations, but do not affect the teaching of the program.

On the pages listed below, the answers given in the Teacher's Guide are numerically correct for the questions in the pupils' text. However, the questions have been changed to include only metric units.

Page 159 3. The answers now refer to cans, not pounds.
Page 162 1, 2, 3, 4. The answers now refer to apples sold, not pounds collected.

Page 192

1. 15 km
2. 74 km
3. 1137 km
4. 187 km
5. 661 km
   589 km

Page 194

1. Yes
2. No
   The unit or measure

Page 196

1a.	1 km	b.	1 cm	c.	1 km
d.	1 m	e.	1 cm	f.	137 cm
g.	1000 cm	h.	same	i.	1 m
2a.	1 km	b.	2 m	c.	1 km
d.	1 km	e.	1 km	f.	100 km
g.	1 km	h.	1 km		
3a.	1 cm	b.	30 mm	c.	2000 mm
d.	1 km	e.	2000 mm	f.	7 m
g.	3 km				

Teacher's Guide
Level 3 Book 3

SRA MATHEMATICS LEARNING SYSTEM TEXT

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8 ADDITION AND SUBTRACTION

before this chapter the learner has—

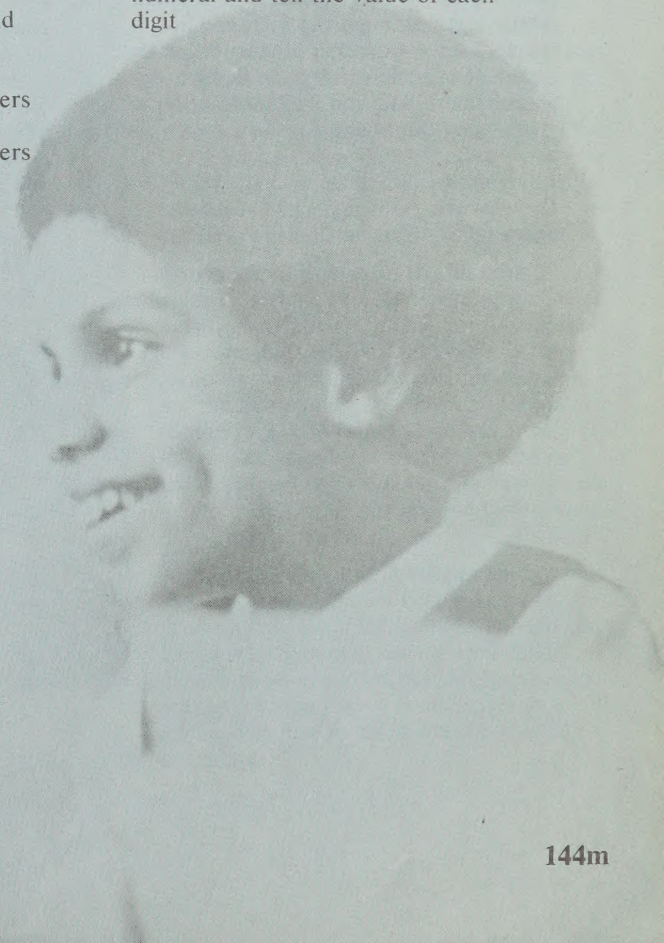
1. Identified the value of each digit of a 3-digit number
2. Mastered finding the sum or difference of any two 2-digit numbers

in chapter 8 the learner is—

1. Correctly reading aloud any 4-digit numeral and telling the value of each digit
2. Comparing two 4-digit numbers and telling which is greater
3. Reviewing finding the sum or difference of any two 2-digit numbers
4. Mastering finding the sum or difference of any two 3-digit numbers
5. Checking subtraction by addition
6. Adding two 4-digit numbers

in later levels the learner will—

1. Add and subtract two 4-digit numbers
2. Read aloud and write any 6-digit numeral and tell the value of each digit



Notes & Things

This addition and subtraction chapter contains a massive review of each skill. There is continued diagnosis and lots of computational practice. The existing knowledge is then expanded to include skills that represent the final level of performance expected of learners at this level. It takes a lot of pupil work to master both addition and subtraction with two 3-digit numbers.

Perhaps some of your pupils have already mastered the content of this chapter, and others will need only a little more practice to get to the mastery level. You probably have a good idea of who will need what. But before you start the work, please look over the whole chapter. Consider the chapter that was just completed. Do you and the children need a break from computation? If so, why not go on to chapter 9 (on measurement) and then come back refreshed and ready to go. Maybe your review of the chapter will encourage you to postpone the chapter even longer or to use only part of the work now. You are free to make any of these decisions, because this chapter is a complete study of addition and subtraction that can stand alone and be profitably used anytime before chapter 14.

The sequential diagnosis and practice of addition and subtraction skills will let you see more and more learners who are willing and able to assess their own skill strengths and weaknesses. Many will begin to assume on their own greater responsibility for individual practice needed to achieve mastery

No special materials are needed. But do make sure that resources are available for any youngster who needs more practice. Be prepared for the precocious children too. Activity programs, puzzles, and games will be necessary if pupils progress independently through this chapter.

goal Think about and explore ideas through a picture clue

page 145 The youngsters need to start thinking about large numbers. The photograph will help. The grain pouring out of a chute will mean one thing to children who know something about the farm and may mean nothing at all to city folks. The first discussion ideas listed will be for the city folks.

Find out if they know that the picture shows shelled corn. *Who knows what corn is? What does it look like? What does the plant look like? How tall is the plant? Where does it grow? Who grows it? When is it ready to be picked? What happens to it after it gets picked?* Have an encyclopedia handy. It will help. Maybe your very capable will be able to do some research.

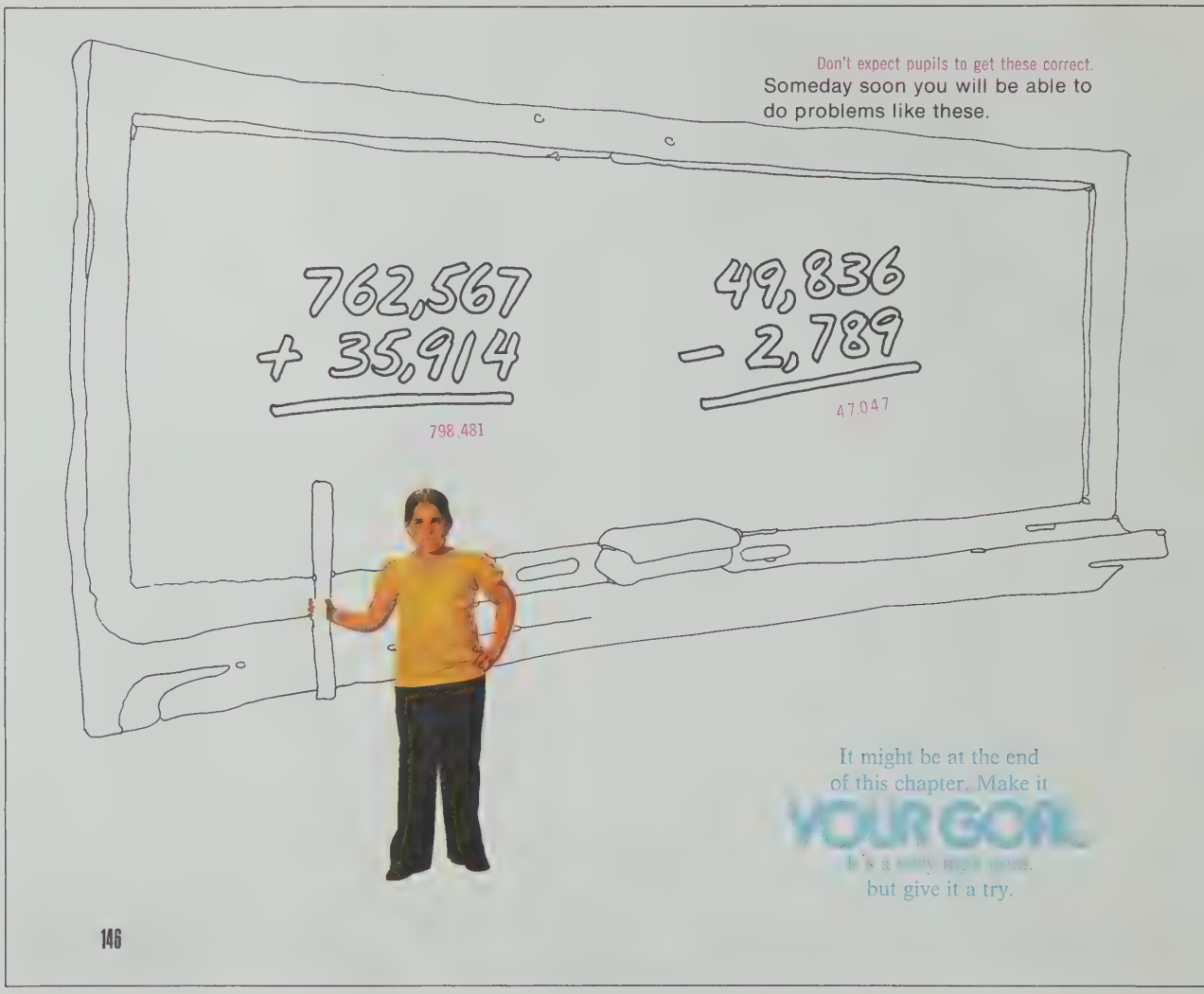
Those youngsters who live in corn country should be asked different questions. Find out if they know whether the corn they eat as a vegetable is the same as the corn in the picture and how big the fields are where corn is planted. (Some rural children will be quick to give the number of acres usually planted in their region.) *How does the farmer decide whether or not he has a good corn crop? How many bushels per acre are expected?*

And then, of course, it is appropriate to end the discussion for both groups by investigating what things corn is used for. Large numbers may not be specifically used in the discussion, but surely everyone will appreciate that it would take an awful lot of corn just to make breakfast food for everyone.

lesson Page 146

goal **Survey**—advanced addition and subtraction skills

page 146 Learning goals give the learner something to work toward. These problems are meant to challenge, not to defeat. Make no effort to force pupils to complete them now. But if any pupil can do these confidently, check him out immediately by doing page 174, rows 3, 4, and 5. Please, for goodness' sake, don't have him do all the work in this chapter. Unnecessary practice is a sure way to get a youngster to hate math.



Talk about these. 1. Can you read these?

five hundred thirty-four **a** 534 **b** 6712 six thousand, seven hundred twelve

eight thousand, nine hundred seven **c** 8907 **d** 12,345 twelve thousand, three hundred forty-five

nineteen thousand, six hundred seventy-eight **e** 19,678 **f** 981,765 nine hundred eighty-one thousand, seven hundred sixty-five

one million **g** 1,000,000 **h** 8,625,394 eight million, six hundred twenty-five thousand, three hundred ninety-four

2. Which numbers above have the digit 0? c, g
Which numbers in the list above have the digit 1? b, d, e, f, g
Can you find a number with the digit—

b, d, or h **a** 2? **b** 3? a, d, or h

a, d, or h **c** 4? **d** 5? a, d, f, or h

b, e, f, or h **e** 6? **f** 7? b, c, e, or f

c, e, f, or h **g** 8? **h** 9? c, e, f, or h

Do these. 3. Can you write a number that does not have any one of the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9? No

4. Every whole number that we use is written with one or more of the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.

a Write the smallest whole number you can think of. 0

b Write the largest number you can read. Answers will vary.

c Do you think there are larger numbers than the one you just wrote? Yes

d When do people need to use large numbers? Answers will vary. Examples: population reports, banks, scientists (space program), government spending

e Find a story in the newspaper that uses a large number.

147

See activity 1, page 178a.

See activity 2, page 178a.

lesson Pages 147, 148, 149, 150, 151

goal **Survey**—ability to read large numbers; investigating our system of numeration.

memo The word **digit** (in problem 2) may need some review. A digit is any numeral from 0 through 9. Could this name have anything to do with counting on fingers? *What different symbols do we need to record how many?* The number 444 is a 3-digit number, but it has one digit used three times. You may be able to have fun with this.

page 147 The first problem will survey pupil ability to read large numbers. Please have this exercise completed orally. Spelling of number words is not as important as place value is right now. If the inability to read large numbers is a weakness, provide additional practice with major emphasis on 4-digit numbers. (See the suggested activity.) Send your Supersleuths out to hunt for examples of large numbers in the real world—newspapers, magazines, science stories, and so on. Your Sleuths may need some help in reading their finds.

A question on zero may arise when discussing problem **4a.** Zero is a whole number. It answers the question, how many?

goal Examining the relationship of the specific digit to place-value position and to quantity

page 148 Place value of a number is an important concept when adding and subtracting with large numbers. Take time to talk about this page.

One 2-digit number has been left out in problem 1. Can anyone identify eleven as the missing number?

Examine each group of numbers separately. *Which number is the greatest? Which is the least? What makes some numbers greater and other numbers less? Are there any clues?*

1. See what you can do with just two digits—0 and 1

a	0—How many?	zero
b	1—How many?	one
c	10—How many?	ten
d	100—How many?	one hundred
e	101—How many?	one hundred one
f	110—How many?	one hundred ten
g	111—How many?	one hundred eleven
h	1000—How many?	one thousand
i	1001—How many?	one thousand, one
j	1010—How many?	one thousand, ten
k	1011—How many?	one thousand, eleven
l	1100—How many?	one thousand, one hundred
m	1101—How many?	one thousand, one hundred one
n	1110—How many?	one thousand, one hundred ten
o	1111—How many?	one thousand, one hundred eleven

Which Is Greatest? 111

Which Is Greatest? 1111

2. How many more do you have if—

a I have 0 and you have 1? 1

b I have 1 and you have 10? 9

c I have 10 and you have 100? 90

d I have 100 and you have 101? 1

e I have 100 and you have 111? 11

f I have 100 and you have 1000? 900

g I have 1000 and you have 1010? 10

h I have 1000 and you have 1111? 111

A number always tells how many. The numeral you write will always use certain digits. The size of a number depends on which digits are used, how many are used, and the order of the digits used.

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See activity 3, page 178a.

1. Which is greater?

   a 0 or 9 **b** 19 or 91 **c** 123 or 321
   d 1432 or 4132 **e** 91,234 or 19,234 **f** 100 or 1000

2. Our number system is neat. We have only the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. We can use each digit as many times as we wish. The order of the digits tells you the value of that digit.

9 means 9 ones (9×1)
90 means 9 tens (9×10)
900 means 9 hundreds (9×100)
9000 means 9 thousands (9×1000)
90,000 means 9 ten-thousands ($9 \times 10{,}000$)
900,000 means 9 hundred-thousands ($9 \times 100{,}000$)
9,000,000 means 9 millions ($9 \times 1{,}000{,}000$)

You can write even larger numbers. As large as you want! But remember—numbers mean something. They tell you "how many."

3. Follow directions. Write the numeral for each.

   a The largest whole number that has only one digit 9
   b The smallest whole number that has only one digit 0
   c The largest whole number that uses the same digit two times 99
   d The largest whole number that uses the same digit three times 999
   e The largest whole number that uses the same digit four times 9999
   f The smallest whole number that uses the same digit four times 1111

149

See activity 4, page 178a.

goal Examining place value

page 149 It's time to hear some oral answers again. In problem 1, listen for reversals of digits when reading the numbers. This may be a visual problem, lack of place-value understanding, or carelessness.

Make the numbers in problem 2 real. Which one of them could tell how many players are on a baseball team? How many people go to this school? How many people live in this town? And so on.

Remind the pupils that the comma does not say "and." It is there to help them read large numbers.

When the directions are understood, problem 3 should be completed independently. Was anyone fooled?

goal Relating the size of a number to specific digits and their place-value position

page 150 The place-value chart can help learners read large numbers correctly. Consider preparing a duplicating master of the place-value chart for use with the next page and for additional practice.

Go back and extend problems 4, 5, and 6 by challenging the youngsters to also name the least possible number. (10,234; 102,345; 1,111,111) Can the Supersleuths determine how much smaller each of the smaller numbers is than the larger one?

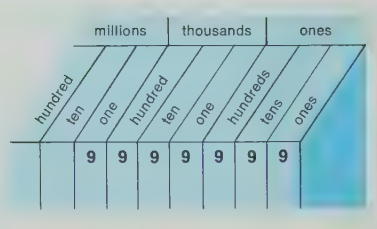
1. Look at the digit 2 in the following numbers.
   - **a** 9872 What is the value of the digit 2 here? 2
   - **b** 9827 What is the value of 2 in this number? 20
   - **c** 9287 What is the value of 2 here? 200
   - **d** 2987 What is the value of the digit 2 here? 2000
2. Which is the largest number in the list above? 9872
   How do you know it's the largest? By comparing the value of the digits
3. Which is the smallest number in the list? 2987
   How do you know? By comparing the value of the digits
4. Use five different digits. Write the largest number you can. 98,765
5. Use six different digits. Write the largest number you can. 987,654
6. If you used only one digit seven times, the largest number you could write would be 9,999,999.

   The same digit was used, but does the digit mean the same thing?

9,999,999

millions | hundred-thousands | ten-thousands | thousands | hundreds | tens | ones

You could put this into a chart.
The chart tells the value of each digit.
It also helps you read large numbers. How?
The headings in the chart tell you what words to say (what value each digit has).



150

See activity 5, page 178a.

1. Your goal is to write the largest number you can.

You can use any of the ten digits except 9.
The digit 9 has already been put into place for each number. BUT for each number you can use a digit only once.

	thousands			ones		
	hundreds	tens	ones	hundreds	tens	ones
a	**9**	?8	?7	?6	?5	?4
b	?8	**9**	?7	?6	?5	?4
c	?8	?7	**9**	?6	?5	?4
d	?8	?7	?6	**9**	?5	?4
e	?8	?7	?6	?5	**9**	?4
f	?8	?7	?6	?5	?4	**9**

2. Change your goal to the smallest number you can.
Use any digits of the ten digits you want, BUT use each only once.

a

1	0	2	3	4	5
?	?	?	?	?	?

or 012,345 (Accept either.)

b How many digits would you use to write the smallest number you know? 1
Which digit would you use? 0
Answers may vary. A student may use a negative number.

***c** Put these numbers in order.
Start with the largest. End with the smallest.

7201 9567 7987 9864 9091 7120

9864, 9567, 9091, 7987, 7201, 7120

goal Writing largest and smallest numbers

page 151 Have fun with this page. It will be a challenge to some learners.

When the page is finished, why not see who can write the largest number. The youngsters are not restricted to **thinking** within the limits of the chart shown on the page. Encourage them to use their imaginations. Get them out of the rut!

things numeral cards 0 through 9; place-value position cards, ones through thousands

Keep the two sets of cards separate. Each set is mixed and placed facedown in a stack. A youngster selects 1 card from each stack and arranges the digit as directed by the place-value position card. He continues until all the place-value cards have been drawn and then reads the number he has arranged.

lesson Pages 152, 153, 154

goal Review of addition and subtraction facts

memo Take advantage of the vocabulary study but don't force memorization of definitions.

page 152 Note that all the problems come in pairs—the same numbers but different operations. Did anyone really know these facts but read the signs carelessly?

Examine examples **3a** and **b** together. *It looks as if adding zero and subtracting zero give the same answer. Is this always true?*

Our number system is organized. The number 1 means the same number of things to every person. Most people understand what the operations of addition and subtraction mean too. When we add two or three numbers, we get a number that is a total. This total is sometimes called the *sum*.

When we subtract one number from another, we find the number that remains. The number that remains is sometimes called the *difference*.

If directions for a set of problems say—
"Add," you know what to do.
"Find the sum," you know what to do.
"Subtract," you know what to do.
"Find the difference," you're ready to go too.
"Compute," then what do you do?
Look at the operation sign; perform that operation on the two numbers given.

COMPUTE

	a	b	c	d	e	f
1.	8 + 7 = 15	8 − 7 = 1	9 + 6 = 15	9 − 6 = 3	7 + 3 = 10	7 − 3 = 4
2.	9 + 8 = 17	9 − 8 = 1	8 + 5 = 13	8 − 5 = 3	7 + 7 = 14	7 − 7 = 0
3.	9 + 0 = 9	9 − 0 = 9	9 + 1 = 10	9 − 1 = 8	1 + 1 = 2	1 − 1 = 0

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In the next pages you will see the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9 over and over and over again. Think about the digit 9.

In 9, the digit 9 signals 9 ones.
In 987, the digit 9 signals 9 hundreds.

You will add and subtract numbers—lots of them. The work won't be hard. But the work is important. You'll work with small numbers and large numbers.
Are you ready to go? Prove it.
Show that you know how to add ones.

	a	b	c	d	e
1.	2 + 6 = 8	4 + 2 = 6	6 + 3 = 9	5 + 3 = 8	2 + 7 = 9
2.	4 + 3 = 7	6 + 4 = 10	4 + 5 = 9	1 + 0 = 1	2 + 5 = 7
3.	6 + 6 = 12	9 + 4 = 13	8 + 6 = 14	7 + 5 = 12	5 + 6 = 11
4.	9 + 5 = 14	8 + 8 = 16	9 + 7 = 16	8 + 4 = 12	7 + 6 = 13
5.	9 + 9 = 18	7 + 8 = 15	7 + 4 = 11	9 + 3 = 12	8 + 3 = 11

153

goal Diagnosis of ability with addition facts

page 153 The addition facts should have been mastered by this time, but here is a chance to make sure. You will be able to identify those youngsters who need additional daily drill and practice on basic facts. Note that rows 1 and 2 contain those facts that have sums of 10 or less, while the sums for the remaining rows are all greater than 10. This is a good place to use the folded-paper technique and write sums only.

goal Diagnosis of ability with subtraction facts

page 154 The subtraction facts should also be mastered by this time. Find out where everyone stands. Rows 1 and 2 contain facts in which the learner must subtract from a number less than 10. For the remaining rows, he must subtract from numbers equal to or greater than 10. Again, use the folded-paper technique and require answers only.

Additional daily practice is a must for those pupils who have not mastered these facts. This is a good opportunity to use capable peer tutors and parents too. You can also go back to the Resource Section for chapters 1 and 5. Provide the most work on the facts each individual needs to practice.

Prove you know how to subtract.
Here are some easy ones.

	a	b	c	d	e	f	g
1	8 − 3 = 5	4 − 2 = 2	8 − 4 = 4	7 − 3 = 4	9 − 7 = 2	7 − 5 = 2	9 − 6 = 3
2	8 − 6 = 2	9 − 2 = 7	6 − 6 = 0	9 − 3 = 6	8 − 2 = 6	6 − 4 = 2	9 − 4 = 5
3	8 − 5 = 3	12 − 7 = 5	11 − 6 = 5	14 − 6 = 8	10 − 5 = 5	13 − 4 = 9	15 − 9 = 6
4	13 − 8 = 5	17 − 9 = 8	12 − 7 = 5	11 − 4 = 7	16 − 8 = 8	10 − 7 = 3	14 − 8 = 6
5	18 − 9 = 9	12 − 6 = 6	16 − 9 = 7	17 − 8 = 9	13 − 5 = 8	15 − 7 = 8	11 − 3 = 8

6 He had 15. He lost 8. How many left? 7

7 She made 14. She broke 9. How many left? 5

8 They bought 13. They returned 7. How many left? 6

154

ADD

	a	b	c	d	e	f	g
1	10 + 10 = 20	20 + 20 = 40	30 + 30 = 60	40 + 40 = 80	50 + 50 = 100	30 + 10 = 40	40 + 20 = 60
2	20 + 30 = 50	50 + 20 = 70	30 + 40 = 70	50 + 30 = 80	40 + 10 = 50	50 + 40 = 90	10 + 50 = 60
3	20 + 10 = 30	90 + 20 = 110	70 + 30 = 100	60 + 40 = 100	90 + 10 = 100	80 + 20 = 100	90 + 90 = 180
4	80 + 40 = 120	70 + 60 = 130	50 + 80 = 130	90 + 30 = 120	80 + 30 = 110	50 + 70 = 120	90 + 40 = 130
5	70 + 40 = 110	50 + 90 = 140	60 + 50 = 110	80 + 60 = 140	90 + 60 = 150	80 + 70 = 150	70 + 90 = 160
6	80 + 50 = 130	60 + 90 = 150	90 + 50 = 140	60 + 60 = 120	70 + 70 = 140	80 + 80 = 160	90 + 90 = 180

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lesson Pages 155, 156

goal Diagnosis of ability and practice in adding two multiples of 10

page 155 There should be a high correlation between ability to learn addition facts and ability to add multiples of 10. Consider having each child complete only one column for diagnosis. Then, for practice, he can complete the row in which he has an error.

Rows 1 and 2 have sums less than 100. The sums are equal to or greater than 100 in the remaining rows.

Note these reasons for lack of success:

- The basic fact involved has not been mastered.
- The concept of operating with multiples of 10 is not understood.

A review of this sequence may help:

8	8 tens	80
+ 3	+ 3 tens	+ 30

Watch particularly for facts with the sum 11. Youngsters sometimes get carried away and make this error:

80	40
+ 30	+ 70
111	111

They probably are not adding the ones column first and then the tens. Some gentle nagging may be in order.

goal Diagnosis of ability and practice in subtracting two multiples of 10

page 156 As with addition, there should be a correlation between mastery of subtraction facts and ability to subtract two multiples of 10. Use the same diagnostic technique as on page 155. This review and practice is preparation for subtraction with renaming.

Subtract

	a	b	c	d	e	f	g
1.	20 − 10 = 10	40 − 20 = 20	60 − 30 = 30	80 − 40 = 40	100 − 50 = 50	110 − 30 = 80	110 − 50 = 60
2.	40 − 10 = 30	60 − 20 = 40	50 − 30 = 20	70 − 20 = 50	70 − 40 = 30	120 − 70 = 50	140 − 60 = 80
3.	80 − 30 = 50	50 − 10 = 40	90 − 40 = 50	60 − 50 = 10	30 − 10 = 20	130 − 40 = 90	150 − 60 = 90
4.	110 − 20 = 90	100 − 30 = 70	100 − 40 = 60	100 − 10 = 90	100 − 20 = 80	110 − 40 = 70	150 − 70 = 80
5.	180 − 90 = 90	120 − 40 = 80	130 − 60 = 70	130 − 80 = 50	120 − 30 = 90	140 − 90 = 50	160 − 90 = 70

156

If you can add $\begin{array}{r} 4 \\ +\ 5 \end{array}$ and $\begin{array}{r} 30 \\ +\ 60 \end{array}$ you know how to add $\begin{array}{r} 34 \\ +\ 65 \end{array}$

tens	ones
3	4
+ 6	5
9	9 ← Add ones first.

Then add tens.

you try this one

tens	ones
4	3
+ 2	5
■	■ ← Add ones first.

Then add tens.

6 8

Practice adding.

	a	b	c	d	e
	tens ones	tens ones	tens ones	tens ones	tens ones
1.	6 2 + 1 5 7 7	2 3 + 3 4 5 7	6 2 + 3 4 9 6	5 1 + 4 3 9 4	2 3 + 1 6 3 9
2.	6 5 + 2 3 8 8	8 7 + 1 2 9 9	3 4 + 2 3 5 7	1 2 + 6 5 7 7	4 5 + 2 2 6 7
3.	3 1 + 3 4 6 5	7 4 + 1 4 8 8	2 5 + 2 1 4 6	1 2 + 8 7 9 9	4 4 + 1 5 5 9

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lesson Pages 157, 158

goal Diagnosis of ability and practice in adding two 2-digit numbers with no renaming

page 157 Review the steps of the algorithm together. Stress adding ones first. Row 1 is sufficient to diagnose ability. Use manipulatives along with problems in row 2. Then check learning with row 3.

Pupils who have no difficulty with row 1 should go directly to page 158.

goal Diagnosis of ability and practice in subtracting two 2-digit numbers with no renaming

page 158 The diagnosis of the subtraction skills parallels that of the addition skills on page 157. Use the same techniques and procedure.

Pupils who work accurately in row 1 go directly to page 159.

If you can subtract $\begin{array}{r} 9 \\ -\,4 \end{array}$ and $\begin{array}{r} 50 \\ -\,20 \end{array}$ you know how to subtract $\begin{array}{r} 59 \\ -\,24 \end{array}$

tens	ones
5	9
− 2	4
■	■
3	5

← Subtract ones first.
Then subtract tens.

you try this one

tens	ones
6	5
− 3	4
■	■
3	1

← Subtract ones first.
Then subtract tens.

PRACTICE SUBTRACTING

	a	b	c	d	e
	tens ones	tens ones	tens ones	tens ones	tens ones
1.	4 9 − 2 8 2 1	3 6 − 1 6 2 0	7 8 − 5 6 2 2	8 6 − 3 5 5 1	9 8 − 6 7 3 1
2.	4 1 − 3 0 1 1	7 9 − 4 9 3 0	5 7 − 2 1 3 6	9 3 − 5 2 4 1	9 6 − 7 1 2 5
3.	6 4 − 4 3 2 1	8 5 − 4 4 4 1	7 4 − 3 1 4 3	6 7 − 2 6 4 1	9 8 − 4 2 5 6

158

If you can add $\begin{array}{r} 4 \\ +\ 7 \end{array}$ and $\begin{array}{r} 50 \\ +\ 30 \end{array}$ you can add $\begin{array}{r} 54 \\ +\ 37 \end{array}$

tens	ones
5	4
− 3	7
1	1
8	
9	1

Add ones first.
Then add tens.
in all

you try this one

tens	ones
6	3
+ 2	8
11	■ ■
8	■
91	■ ■

Then add tens.
in all

(Let the pupils use the short form if they want to.)

Practice.

	a	b	c	d	e
	tens ones	tens ones	tens ones	tens ones	tens ones
1.	5 2 + 2 9 1 1 7 8 1	4 4 + 3 8 1 2 7 8 2	6 3 + 2 7 1 0 8 9 0	6 5 + 1 9 1 4 7 8 4	7 4 + 1 6 1 0 8 9 0
2.	4 5 + 2 7 1 2 6 7 2	3 6 + 2 5 1 1 5 6 1	7 3 + 2 9 1 2 9 10 2	4 4 + 4 9 1 3 8 9 3	5 7 + 3 4 1 1 8 9 1

3. Everyone was collecting cans to be recycled.
Here is a chart of how many pounds were collected each day for five days.
Figure out how much was collected each day.

Pounds collected	Monday	Tuesday	Wednesday	Thursday	Friday
Morning	53	26	45	38	59
Afternoon	18	39	47	44	32
	11 6 71	15 5 65	12 8 92	12 7 82	11 8 91

159

lesson Pages 159, 160, 161, 162

goal Diagnosis of ability and practice in adding two 2-digit numbers with renaming

memo Please don't force a learner to use this long form if he wants to use the paper-saving form.

page 159 Review the algorithm together. Use the first row for diagnostic purposes.

Youngsters who are successful with row 1 should go on to the word problem at the bottom of the page while you work with the others.

Individuals who have difficulty need your help. Have them explain the steps to you. This will help you identify faulty thinking. Use manipulatives. Check performance with row 2.

goal Practice with the short algorithm for adding two 2-digit numbers with renaming

page 160 The learner should be free to use the algorithm with which he feels most comfortable and can work most successfully.

This page provides additional practice for the youngster who uses the long algorithm. Two rows of problems are enough to diagnose the learner's ability to compute with the short algorithm.

Continue to use the long algorithm to tutor those pupils who have difficulty with renaming.

Pupils who have mastered this skill should go on to extension page 162 and work independently.

It's silly to use up extra space if you don't need to,
when you are adding numbers that need to be renamed.

Review the two ways of writing problems.

Here's one way.

tens	ones
4	7
+ 2	8
1	5
6	
7	5

Here's another way.

tens	ones
1	
4	7
+ 2	8
7	5

These are the same problem.
They have the same answer.
You decide which way is better for you.

Copy and add these. Write the computation your way.

	a	b	c	d	e	f	g
1.	59 + 19 = 78	28 + 58 = 86	37 + 47 = 84	25 + 18 = 43	16 + 37 = 53	67 + 14 = 81	49 + 23 = 72
2.	37 + 13 = 50	17 + 78 = 95	25 + 66 = 91	38 + 56 = 94	26 + 29 = 55	67 + 19 = 86	54 + 29 = 83
3.	35 + 37 = 72	26 + 36 = 62	38 + 15 = 53	49 + 12 = 61	26 + 48 = 74	48 + 39 = 87	56 + 35 = 91

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Sometimes you must rename in subtraction, too.

tens	ones	
2	14	Rename first.
~~3~~	~~4~~	
− 1	9	
1	5	← Then subtract ones.
		← Then the tens.

you try this one

tens	ones	
4	11	Rename first.
~~5~~	~~1~~	
− 1	7	
■	■	← Then subtract ones.
		← Then tens.
3	4	

Copy and subtract. Renaming is needed in every problem.

	a	b	c	d	e	f	g
1.	3 12 ~~4~~ ~~2~~ − 1 6 2 6	5 13 ~~6~~ ~~3~~ − 1 7 4 6	6 15 ~~7~~ ~~5~~ − 1 9 5 6	4 10 ~~5~~ ~~0~~ − 2 3 2 7	2 14 ~~3~~ ~~4~~ − 1 6 1 8	7 16 ~~8~~ ~~6~~ − 2 7 5 9	3 17 ~~4~~ ~~7~~ − 1 9 2 8
2.	7 16 ~~8~~ ~~6~~ − 3 9 4 7	6 12 ~~7~~ ~~2~~ − 4 7 2 5	3 15 ~~4~~ ~~5~~ − 2 6 1 9	8 18 ~~9~~ ~~8~~ − 7 9 1 9	7 13 ~~8~~ ~~3~~ − 4 5 3 8	6 17 ~~7~~ ~~7~~ − 5 8 1 9	5 15 ~~6~~ ~~5~~ − 4 8 1 7
3.	5 12 ~~6~~ ~~2~~ − 3 4 2 8	2 13 ~~3~~ ~~3~~ − 2 9 4	8 12 ~~9~~ ~~2~~ − 1 5 7 7	4 13 ~~5~~ ~~3~~ − 4 6 7	3 14 ~~4~~ ~~4~~ − 3 9 5	6 16 ~~7~~ ~~6~~ − 4 8 2 8	7 11 ~~8~~ ~~1~~ − 6 7 1 4

161

goal Diagnosis of ability and practice in subtracting two 2-digit numbers with renaming

page 161 Review first; then diagnose ability with the first row.

Pupils who perform well can go directly to page 162. Those who have errors need you. Work row 2, using manipulatives. Ask questions. Get the pupils to verbalize their thinking, but please don't expect any technical vocabulary. Encourage the completion of the renaming as the first step in subtraction.

goal Application of and practice in adding and subtracting 2-digit numbers

page 162 If this page has not already been used as an extension activity, you may wish to complete problems 1 through 4 as a group project.

It's time for a change of pace. Can anyone build new wheels by changing the numbers?

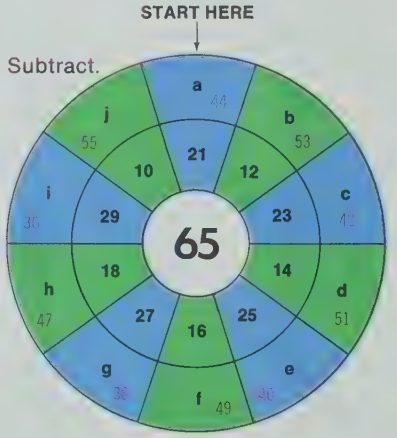
The boys said they could collect more scrap paper to sell than the girls. The girls knew better. The paper drive was to come to a close in one week. Here is their record for the week.

Pounds of paper	Monday	Tuesday	Wednesday	Thursday	Friday
Boys	97	74	96	85	93
Girls	92	86	96	96	75

1. Who brought in the most each day? How much more?
Monday: boys, 5 lb. Tuesday: girls, 12 lb. Wednesday: same amount Thursday: girls, 11 lb. Friday: boys, 18 lb.
2. How many pounds in all did the boys collect that week? 445
3. How many pounds in all did the girls collect that week? 445
4. Did the boys or the girls collect more paper?
Both collected the same number of pounds of paper.

5. Add.

6. Subtract.



If you can add $\begin{array}{r} 2 \\ +\,6 \end{array}$ and $\begin{array}{r} 30 \\ +\,90 \end{array}$ you can add $\begin{array}{r} 32 \\ +\,96 \end{array}$.

hundreds	tens	ones	
	3	7	
+	9	6	
	1	3	Add ones.
1	2		Add tens.
1	3	3	in all

Some can be written all on one line.

hundreds	tens	ones	
	6	1	
+	4	5	
1	0	6	Add ones.
			Add tens.

Practice.

	a	b	c	d	e
1.	$\begin{array}{r} 73 \\ +\,45 \\ \hline 118 \end{array}$	$\begin{array}{r} 83 \\ +\,46 \\ \hline 129 \end{array}$	$\begin{array}{r} 96 \\ +\,63 \\ \hline 159 \end{array}$	$\begin{array}{r} 75 \\ +\,64 \\ \hline 139 \end{array}$	$\begin{array}{r} 82 \\ +\,90 \\ \hline 172 \end{array}$
2.	$\begin{array}{r} 92 \\ +\,77 \\ \hline 169 \end{array}$	$\begin{array}{r} 83 \\ +\,74 \\ \hline 157 \end{array}$	$\begin{array}{r} 92 \\ +\,54 \\ \hline 146 \end{array}$	$\begin{array}{r} 93 \\ +\,46 \\ \hline 139 \end{array}$	$\begin{array}{r} 83 \\ +\,31 \\ \hline 114 \end{array}$

The bakery is a great place to go. The baker makes so many things.

3. One day he made 96 loaves of white bread and 84 loaves of dark bread. How many loaves did he make? 180

4. He made 96 chocolate and 48 white cakes. How many cakes? 144

5. He made 36 pans of pecan rolls and 72 pans of Danish rolls. How many pans of rolls did he make? 108

6. You tell how many cookies he might have made that day.
Accept reasonable answers. Possible range: 120 to 1200

things for each group: egg carton, crayon, 2 rubber washers

Write 2- and 3-digit numerals in the cups of the egg carton. In turn, each person tosses the rubber washers into the cups one at a time. This sum is his score. Winner of the round is the person with the highest score.

things 100 counters

Try to arrange 3 counters so that they form a square array. (Not possible) Try 4 counters. (2-by-2 array) The goal is to find all the numbers less than 100 that can be made into a square array. (4, 9, 16, 25, 36, 49, 64, 81, 100)

lesson Pages 163, 164

goal **Progress Check**—adding two 2-digit numbers with renaming

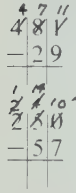
page 163 Both the long and the short algorithms are there for review. Let everyone do his own thing. Use rows 1 and 2 for the Progress Check.

The rest of the page is reserved for independent learners.

goal **Progress Check** – subtracting numbers with renaming

memo Hundreds have been included in each problem. This parallels the work on page 163. Notice that when renaming is required, the steps are identical to subtracting two 2-digit numbers.

Watch for the individualist who renames in this way:



Renaming 48 tens as 47 tens is a great idea, but watch out for

Here the renaming can get pretty messy. If the renaming is completed correctly, give praise and your blessings. But watch for problems later that result more from poor penmanship than lack of knowledge. Problem **3b** will let you check on this.

page 164 There should be no trouble. Since the problems that involve renaming are about evenly split with those that do not, use only rows 1 and 2 for the Progress Check.

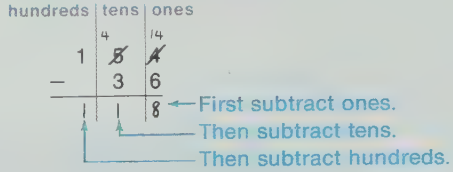
Determine the reason for the learner's failure.

- Careless subtraction errors
- Has difficulty determining when to rename
- Has not mastered the steps involved in renaming

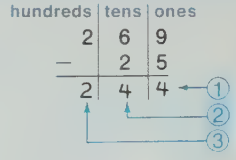
Provide teacher or able-peer tutoring. Manipulatives may help. Then use row 3 to recheck these youngsters.

Some digits turned up in the hundreds place in the last page with addition. Switch to subtraction. Try these. Sometimes you have to rename. Sometimes you don't.

Here you do.



Here you don't.



Copy and subtract.

	a	b	c	d	e
1.	341 − 36 = 305	256 − 48 = 208	199 − 29 = 170	148 − 16 = 132	267 − 49 = 218
2.	379 − 62 = 317	293 − 38 = 255	471 − 62 = 409	576 − 75 = 501	481 − 29 = 452
3.	116 − 13 = 103	260 − 57 = 203	391 − 43 = 348	134 − 15 = 119	199 − 31 = 168

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things egg cartons from activity on page 163, rubber washers

Use the same materials and switch the operation to subtraction. This is an appropriate time to have the winner be the one with the lowest score.

See activity 6, page 178b.

If you can add $\begin{array}{r} 3 \\ +\ 4 \end{array}$ and $\begin{array}{r} 50 \\ +\ 20 \end{array}$ and $\begin{array}{r} 600 \\ +\ 100 \end{array}$ you can add $\begin{array}{r} 653 \\ +\ 124 \end{array}$

hundreds	tens	ones	
6	5	3	
+ 1	2	4	
7 ?	7	7	← Add ones first.
			Then add tens.
			Then add hundreds.

you try this one

hundreds	tens	ones	
5	4	6	
+ 4	2	3	
■	■	■	← Add ones.
			Add tens.
			Add hundreds.
9	6	9	

What would happen in a problem like this?

hundreds	tens	ones	
1			
2	7	3	
+	8	1	
■	■	■	← Add ones. Any tens to rename?
			Add tens. Any hundreds to rename? You know what to do.
			Add hundreds and you have found the sum.
3	5	4	

Practice on these.

	a	b	c	d	e
1.	1 7 8 5 + 7 3 8 5 8	1 2 1 4 + 9 5 3 0 9	1 3 6 7 + 8 2 4 4 9	1 4 7 9 + 3 0 5 0 9	1 6 9 + 2 3 1 9 2
2.	1 6 5 2 + 1 6 1 8 1 3	1 4 7 7 + 2 3 2 7 0 9	1 5 9 8 + 1 9 0 7 8 8	1 1 5 1 + 1 5 5 3 0 6	1 1 9 7 + 2 8 0 4 7 7

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lesson Pages 165, 166, 167, 168, 169

goal Diagnosis of ability and practice in adding 2- and 3-digit numbers with renaming

page 165 Renaming tens to hundreds is the focus of this page. Stress adding ones first, then tens, and finally hundreds.

The goal is for each pupil to demonstrate mastery in addition. Check performance on the first problems and group any youngsters who need more help. (Cross your fingers—success for everyone!)

goal Diagnosis of and practice in renaming twice in the addition of two 3-digit numbers

page 166 It is essential that the example be reviewed. It gives you the chance to reemphasize that ones are added first, then tens, then hundreds.

All ten problems are necessary for diagnosis. Row 2 focuses on renaming to hundreds, while row 3 involves renaming twice. Note particularly example **3c.** The sum has 4 digits. Page 167 will provide additional practice on this type of problem.

You are getting good in addition.

hundreds	tens	ones
	1	
1	3	4
+	5	7
?	?	?
1	9	1

Is there anything hard about this problem?
Compute the sum.
What do you do first? Then what? Anything more?
Add the ones. Add tens. Yes—add hundreds.

You shouldn't have any trouble with these either.
Copy and then add.

	a	b	c	d	e
1	1 237 +129 366	11 389 +315 704	1 478 +109 587	11 138 +175 313	11 169 +561 730

All you have to do is take one step at a time.
First add the ones, then add the tens, and finally
add the hundreds. Show that you know how. Add these.

	a	b	c	d	e
2	1 567 +151 718	1 453 +474 927	1 182 +382 564	11 275 +369 644	1 346 +507 853
3	11 368 +194 562	11 176 +444 620	11 998 +164 1162	11 446 +375 821	11 139 +283 422

166

hundreds	tens	ones	
1	1		
9	6	2	
+ 3	6	8	
■	■	■	← Add ones. Any tens to rename?
			← Then add tens. Any hundreds? Put them in the right place.
			← Add hundreds. Any thousands? Any problems?
13	3	0	

You can add just about any two whole numbers

Small ones or large ones should cause you no trouble. Take one step at a time. Add ones first. Rename if necessary. Then add tens. Rename if necessary. And just keep going until you have finished.

Practice.

	a	b	c	d	e
1	1 9 6 9 + 4 7 0 14 3 9	1 1 7 3 8 + 5 6 3 13 0 1	8 4 1 + 5 0 6 13 4 7	1 1 4 8 7 + 8 2 9 13 1 6	1 1 6 4 6 + 9 5 4 16 0 0
2	1 7 3 1 + 7 7 7 15 0 8	1 9 3 0 + 8 8 5 18 1 5	1 8 3 3 + 4 0 9 12 4 2	1 1 3 6 4 + 8 8 8 12 5 2	1 9 4 8 + 2 7 0 12 1 8
3	1 9 8 6 + 7 6 0 17 4 6	1 1 4 6 5 + 6 9 8 11 6 3	1 1 9 4 4 + 2 7 9 12 2 3	1 7 4 6 + 8 0 5 15 5 1	1 1 5 9 4 + 9 8 6 15 8 0

goal Practice in adding two 3-digit numbers with renaming

page 167 The problems on this page represent the limit of mastery expectation for this level. All problems should be completed for practice as well as for diagnosis.

Several factors could be the cause for the learner's lack of success.

- Has incomplete mastery of basic addition facts
- Has difficulty determining when to rename
- Has not mastered the steps involved in renaming

Watch especially for the pupil who automatically renames twice whether or not this step is necessary.

7 3 1	8 4 1
+ 7 7 7	+ 5 0 6
15 1 8	14 5 7

Provide the specific type of tutoring that is necessary.

goal Finding computational errors

page 168 Finding someone else's error is an important step to recognizing one's own error. Challenge your pupils to be good detectives—Supersleuths!

This page could be very frustrating for any pupils who are in real trouble. You may want them to ignore the answers on the paper and copy and complete the problems only. Then they can compare their own answers to the ones in the book. A peer tutor could work to help resolve any differences in answers effectively.

Look for mistakes on this page.

Is there something wrong in this problem? No

hundreds	tens	ones
5	7	4
+ 1	9	8
7	7	2

Is there something wrong in this problem? Yes

hundreds	tens	ones
6	8	5
+ 1	7	6
~~8~~	~~5~~	~~1~~
8	6	1

Find the problems with the wrong answer. Make the answer correct.

	a	b	c	d
1.	125 + 879 1004	504 + 168 672	522 + 69 ~~581~~ 591	979 + 679 1658
2.	578 + 859 ~~1337~~ 1437	586 + 568 ~~1144~~ 1154	478 + 965 1443	967 + 868 ~~1836~~ 1835
3.	549 + 987 1536	856 + 879 ~~1725~~ 1735	596 + 657 ~~1153~~ 1253	476 + 995 1471

Thousands should have a place to call its own.
It is a big word. A chart would look like this:

Thousands	hundreds	tens	ones

THAT TAKES TOO MUCH SPACE!

Use a capital T to stand for thousands.

	T	h	t	o
		7	3	5
+		5	6	4
	1	2	9	9

WHY NOT GO ALL THE WAY?

Try adding thousands too. Look at these examples.

A

T	h	t	o
4	0	0	0
+ 5	0	0	0
9	0	0	0

B

T	h	t	o
3	0	0	2
+ 2	0	0	5
5	0	0	7

C

T	h	t	o
2	0	1	3
+ 4	0	8	6
6	0	9	9

D

T	h	t	o
2	5	6	3
+ 1	3	1	1
3	8	7	4

E

T	h	t	o
5	9	0	4
+ 2	0	7	3
7	9	7	7

That doesn't look so hard. Try some. Find out for yourself.

1.

a

T	h	t	o
1	0	0	6
+ 1	0	0	3
2	0	0	9

b

T	h	t	o
8	0	1	5
+ 1	0	8	4
9	0	9	9

c

T	h	t	o
2	1	6	1
+ 5	1	0	2
7	2	6	3

d

T	h	t	o
2	1	1	2
+ 3	8	5	7
5	9	6	9

e

T	h	t	o
3	4	7	6
+ 2	5	1	2
5	9	8	8

The next ones have some renaming. But there are no new rules.

2.

T	h	t	o
		1	
1	8	2	5
+ 2	1	6	9
3	9	9	4

T	h	t	o
		1	
2	0	2	9
+ 4	1	1	4
6	1	4	3

T	h	t	o
		1	
1	1	2	5
+ 4	6	0	8
5	7	3	3

T	h	t	o
		1	
1	3	0	9
+ 7	0	6	7
8	3	7	6

T	h	t	o
		1	
4	7	3	8
+ 1	2	5	6
5	9	9	4

goal Extension of addition skills to adding two 4-digit numbers with and without renaming

page 169 These problems are not as bad as they look and should be an interesting but reasonable challenge. If the addition facts are mastered and the youngsters know when and how to rename, the number of digits in each number should make no difference.

Use an "I'll bet you know all there is to know to be able to do these big problems!" approach while discussing the first example and completing examples **A** through **D** together. Then encourage the pupils to show how well they can add.

lesson Pages 170, 171, 172

goal Diagnosis of ability and practice in subtracting two 3-digit numbers with renaming

page 170 The problems on this page represent the limit of mastery expectation for this level. The emphasis is on renaming. A zero has been included in the last problem even though zero in subtraction will not be examined thoroughly until page 172.

Row 1 is sufficient to identify those who need your help. Use row 2 to work with youngsters who have trouble.

You can probably subtract large numbers, too.
Take just one step at a time. Rename when necessary.

hundreds	tens	ones
6	7	8
− 3	2	4
3	5	4

hundreds	tens	ones
	5	12
5	~~6~~	~~2~~
− 3	1	5
2	4	7

hundreds	tens	ones
3	15	
~~4~~	~~5~~	9
− 2	7	3
1	8	6

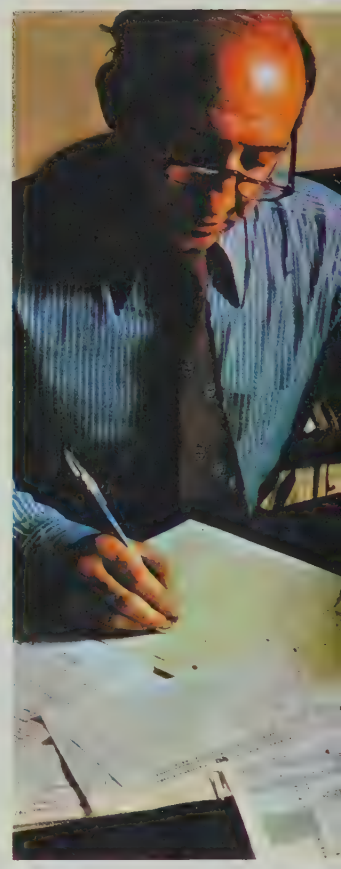
Are you ready for a very, very hard one?

hundreds	tens	ones
2	13	16
~~3~~	~~4~~	~~6~~
− 1	7	9
1	6	7

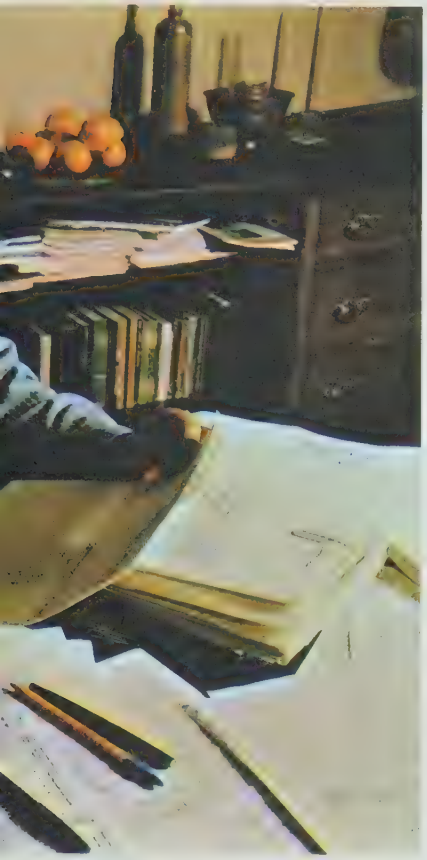
First the ones. Do you have to rename? **Do it!**
Now the tens. Do you have to rename? **Do it!**
Now the hundreds. Any trouble?

Subtract. Don't panic. Take one step at a time.

	a	b	c	d	e
1.	7 17 5 ~~8~~ ~~7~~ − 2 7 9 3 0 8	6 11 12 ~~7~~ ~~2~~ ~~2~~ − 2 7 8 4 4 4	4 14 13 ~~5~~ ~~5~~ ~~3~~ − 1 8 6 3 6 7	8 13 17 ~~9~~ ~~4~~ ~~7~~ − 4 5 8 4 8 9	6 14 16 ~~7~~ ~~5~~ ~~6~~ − 3 5 7 3 9 9
2.	2 12 11 ~~3~~ ~~3~~ ~~1~~ − 2 5 4 7 7	8 11 18 ~~9~~ ~~2~~ ~~8~~ − 1 2 9 7 9 9	5 12 11 ~~6~~ ~~3~~ ~~1~~ − 1 3 8 4 9 3	8 11 10 ~~9~~ ~~2~~ ~~0~~ − 2 5 6 6 6 4	7 10 10 ~~8~~ ~~1~~ ~~0~~ − 5 2 1 2 8 9



170



Some people like to make sure their answers are correct.
There is a way to check subtraction problems.

$\begin{array}{r} 14 \\ -\ 9 \\ \hline 5 \end{array}$ Correct? → Check $\begin{array}{r} 5 \\ +\ 9 \\ \hline 14 \end{array}$ You subtracted 9, so when you check you put it back. Is the answer correct? Yes

$\begin{array}{r} 93 \\ -\ 17 \\ \hline 75 \\ 76 \end{array}$ Correct? → Check $\begin{array}{r} 75 \\ +\ 17 \\ \hline 82 \end{array}$ $\begin{array}{r} 76 \\ +\ 17 \\ \hline 93 \end{array}$ Yes
SOMETHING IS WRONG?
Where is the error?
Subtraction of the ones and addition of the ones in the check

$\begin{array}{r} 125 \\ -\ 96 \\ \hline 29 \end{array}$ Correct? → Check $\begin{array}{r} 29 \\ +\ 96 \\ \hline 126 \end{array}$ $\begin{array}{r} 29 \\ +\ 96 \\ \hline 125 \end{array}$ It doesn't check
WHAT'S WRONG NOW?
Where is the error?
Addition of the ones

Check these subtraction problems on your paper.
Correct the error if there is one.

1. $\begin{array}{r} 435 \\ -\ 167 \\ \hline 268 \end{array}$ Check $\begin{array}{r} 268 \\ +\ 167 \\ \hline 435 \end{array}$?

2. $\begin{array}{r} 945 \\ -\ 196 \\ \hline 849 \\ 749 \end{array}$ Check $\begin{array}{r} 849 \\ +\ 196 \\ \hline 1045 \end{array}$ $\begin{array}{r} 749 \\ +\ 196 \\ \hline 945 \end{array}$?

3. $\begin{array}{r} 922 \\ -\ 138 \\ \hline 784 \end{array}$ Check $\begin{array}{r} 784 \\ +\ 138 \\ \hline 922 \end{array}$?

4. $\begin{array}{r} 810 \\ -\ 236 \\ \hline 574 \end{array}$ Check $\begin{array}{r} 574 \\ +\ 236 \\ \hline 810 \end{array}$?

Does a check guarantee your answer is correct?
No, but if it checks, the answer is probably correct. There could be an error in the check

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goal Checking subtraction computation with addition

page 171 To find that **something is wrong** is not sufficient. Stress finding each error **and** correcting it. Help the pupils to see that the mistake can be in the check, **or** it can be in the subtraction example. If the error is not corrected, the check is useless. Youngsters who see the addition-subtraction relation often write the numbers in their proper position without performing the addition operation. *Is this really checking? Does it help discover the possibility of an error?*

You'll want to talk about this together, but problems 1 through 4 are independent work.

goal Examining zero in addition and subtraction

page 172 Sharing ideas will help everyone with the ideas on this page. We are not introducing negative numbers here. **Praise** the youngster who knows $0 - 5$ equals a number less than zero, but do not stress this idea for the present. At this point, a subtraction such as $0 - 5$ in a 2-digit problem should be a signal for the learner to rename.

Zero in addition does not seem to cause a problem. In subtraction, however, many pupils do run into trouble. Watch particularly for those who combine addition and subtraction in the same problem. For example:

$$\begin{array}{r} 500 \\ -149 \\ \hline 449 \end{array}$$

Use manipulatives with these youngsters to help them see their error.

Encourage the pupils to make a personal decision about which of two renaming techniques they will use. They should stick with the one they choose.

This form is the same form used for any renaming problem, but there are many chances for error in the notation.

$$\begin{array}{r} {\scriptstyle 9\ \ 4\,10\,10} \\ \not{5}\not{0}\not{0} \\ -149 \end{array}$$

This is a safer technique, but if the child does not think of 500 as 50 tens and 0 ones, he will not use this idea successfully.

$$\begin{array}{r} {\scriptstyle 4\ 9\ 10} \\ \not{5}\not{0}\not{0} \\ -149 \end{array}$$

The digit 0 can be a troublemaker if you let it.

$$\begin{array}{r} 5 \\ +\ 0 \\ \hline ? \end{array} \qquad \begin{array}{r} 5 \\ -\ 0 \\ \hline ? \end{array} \qquad \begin{array}{r} 0 \\ +\ 5 \\ \hline ? \end{array} \qquad \begin{array}{r} 0 \\ -\ 5 \\ \hline \end{array}$$

That's a silly problem. There is no whole-number answer.

1. Don't let 0 fool you in addition. Prove that you know how to compute with oodles of zeros.

a	b	c	d	e
60	500	700	1001	90,000
+ 40	+ 200	+ 300	+ 2010	+ 10,000
100	700	1000	3011	100,000

WOW!

2. There are many ways to be master over 0 in subtraction. Look at one problem computed in two ways. Take your pick. There may be a way that you like even better.

$$\begin{array}{r} {\scriptstyle 9} \\ {\scriptstyle 4\,\not{5}\,10} \\ \not{5}\not{0}\not{0} \\ -\ 149 \\ \hline 351 \end{array}$$

THINK

I must rename to have 10 ones.
But there are 0 tens. So I rename
5 hundreds to get the 10 tens.
Now I can rename a ten.
And I'm ready to subtract!

OR you could think about renaming 50 tens to 49 tens so you can be ready to subtract. The results are the same.

$$\begin{array}{r} {\scriptstyle 4\ 9\ 10} \\ \not{5}\not{0}\not{0} \\ -\ 149 \\ \hline 351 \end{array}$$

Pick the way that's best for you.
Practice on these subtraction problems.
Check your answers.

	a		b		c		d		e	
3.	200	149	400	114	700	126	2000	361	3500	1022
	− 51	+ 51	− 286	+ 286	− 574	+ 574	− 1639	+ 1639	− 2478	+ 2478
	149	200	114	400	126	700	361	2000	1022	3500

These are harder. Try them.

172

lesson Pages 173, 174

goal Practice with skills developed in the chapter

page 173 Order of the numbers in subtraction problems **is** important. This is not true for addition. You may want to talk about this before making an assignment.

Use rows 1, 2, and 3 to meet individual needs for additional practice. Assign as much or as little practice as is needed. Everyone should have fun with the last three problems.

1. Subtract 123 from each of these numbers.

a	**b**	**c**	**d**	**e**	**f**
123	132	213	231	312	321
0	9	90	108	189	198

2. Subtract 456 from each of these numbers.

a	**b**	**c**	**d**	**e**	**f**
456	465	546	564	645	654
0	9	90	108	189	198

3. Subtract 111 from each of these numbers.

a	**b**	**c**	**d**	**e**	**f**
200	201	210	211	221	222
89	90	99	100	110	111

4. Arrange each set of three numbers so that you have a correct addition problem. Rearrange the same three numbers so that you have a correct subtraction problem. See answers below.

a 123, 456, 579 **b** 627, 258, 369

c 147, 936, 789 **d** 975, 654, 321

e 1308, 951, 357 **f** 239, 985, 1224

5. What is the largest number you can add to or subtract from 585 and get an answer of 585? 0

6. Find two addition problems that each have a sum of 111. Answers will vary.
Now find two subtraction problems that each have a difference of 111. Answers will vary.
Are any numbers the same except the number 111 in your four problems? Probably not
Can you find more problems that have an answer 111? Try it.

4a $123 + 456 = 579$ or $456 + 123 = 579$, $579 - 123 = 456$ or $579 - 456 = 123$

4b $258 + 369 = 627$ or $369 + 258 = 627$, $627 - 258 = 369$ or $627 - 369 = 258$

4c $147 + 789 = 936$ or $789 + 147 = 936$, $936 - 147 = 789$ or $936 - 789 = 147$

4d $321 + 654 = 975$ or $654 + 321 = 975$, $975 - 654 = 321$ or $975 - 321 = 654$

4e $357 + 951 = 1308$ or $951 + 357 = 1308$, $1308 - 951 = 357$ or $1308 - 357 = 951$

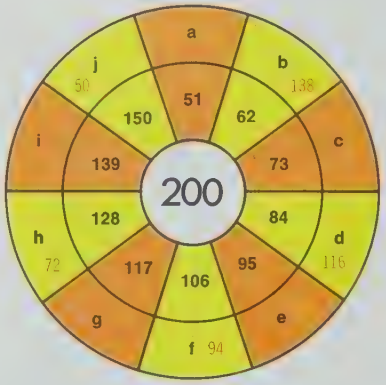
4f $239 + 985 = 1224$ or $985 + 239 = 1224$, $1224 - 239 = 985$ or $1224 - 985 = 239$

goal Practice with skills developed in the chapter

page 174 You may want to start with rows 3, 4, and 5. These combine the skills that have been developed in the chapter. You can use these as a diagnostic check to identify those pupils who need additional tutoring before the Checkout. Independent learners can then work on exercises 1 and 2. Challenge these supercomputers to make up additional wheels. They can be exchanged and then completed.

1. Subtract.

START HERE



2. Add.

- **a** 149 + 51 = ? 200
- **b** 138 + 62 = ? 200
- **c** 127 + 73 = ? 200
- **d** 116 + 84 = ? 200
- **e** 105 + 95 = ? 200
- **f** 94 + 106 = ? 200

Something funny?

Where did these addition problems come from?

These problems check the answers to the wheel problems.

Compute. Watch out! There are both addition and subtraction.

	a	b	c	d	e	f
3.	347 + 273 = 620	422 + 316 = 738	29 − 11 = 18	708 + 49 = 757	596 − 485 = 111	914 + 86 = 1000
4.	370 + 158 = 528	293 − 127 = 166	937 − 768 = 169	351 + 26 = 377	456 + 654 = 1110	679 − 669 = 10
5.	868 − 379 = 489	379 + 588 = 967	667 − 378 = 289	620 − 155 = 465	673 + 127 = 800	2540 − 1539 = 1001

Mr. Travis worked in a manufacturing plant.
His job was to make sure machine parts were O.K.
He had to make out a weekly report. He told how many parts had been made and how many parts were not good enough to send out. The bad parts were called rejects. Here is his report.

Pieces made	Monday	Tuesday	Wednesday	Thursday	Friday
First shift	596	564	530	585	497
Second shift	432	323	469	404	512
Totals:	1028	887	999	989	1009
Pieces rejected					
First shift	51	46	30	49	9
Second shift	25	20	28	10	10
	76	66	58	59	19
O.K to send out:	952	821	941	930	990

1. How many pieces were made each day of the week? See chart above
2. What was the total number of pieces made that week? 4912
3. How many pieces were rejected each day of the week? See chart above.
4. What was the total number of pieces rejected that week? 278
5. How many pieces were O.K. to send out each day? See chart above
6. What was the total number of pieces O.K. to send out that week? 4634
7. Which was the "best" day of the week for manufacturing parts? Friday
   How did you decide what made the best day of the week?
   Answers will vary. Examples: Friday had the least number of pieces rejected. Friday had the second-greatest total of pieces made. Friday had the greatest number of pieces that were O.K.

175

lesson Pages 175, 176, 177

goal Application of addition and subtraction skills

page 175 There's a lot of computation here! How about using this page as a class project? It may require more than one day. You may wish to assign specific computations to specific pupils, according to ability. Several pupils, each computing the same problem, can serve as a check on accuracy. Some of the computations involved provide an excellent opportunity to introduce the office calculator.

goal Finding errors in addition and subtraction problems

memo All the common errors in subtraction are to be found on Fred's paper. The youngsters will be able to find the errors, but may have a hard time finding the right words to describe the type of error made. Please don't force that discussion.

page 176 The directions tell the pupils to work the problems. Make sure they copy just the problems. Don't let them be confused by poor Fred's answers. Have the youngsters compute the correct answers independently. Encourage them to check their subtraction answers by addition.

Each pupil should compare his answers to Fred's. Let them show Fred's errors on the chalkboard. Pointing a finger can save lots of words. Praise those who are able to spot the errors. Others will learn from the ideas shared.

Sometimes it is fun to let the pupils give the teacher a test. Have several children write problems on the board for you to solve. Deliberately make some mistakes. Let them correct your work. Let them tutor you.

Skill: Computing with whole numbers

Fred still forgets more than he remembers. One day he forgot how to add *and* he forgot how to subtract. Here is his paper. He made so many mistakes. Find his mistakes. Work the problem right. Be ready to explain to Fred what he forgot.

1. 75 + 16 = 81	2. 275 + 59 = 234	3. 367 + 136 = 493	4. 32 − 16 = 24
5. 351 − 75 = 324	6. 450 − 196 = 266	7. 342 − 156 = 296	8. 876 + 159 = 1611

He should have subtracted 6 from 12, not 2 from 6.

He wrote 1 in the ones column and 5 at the top of the tens column. They should be reversed. He did the same thing for tens and hundreds.

2 hundreds, 10 tens.

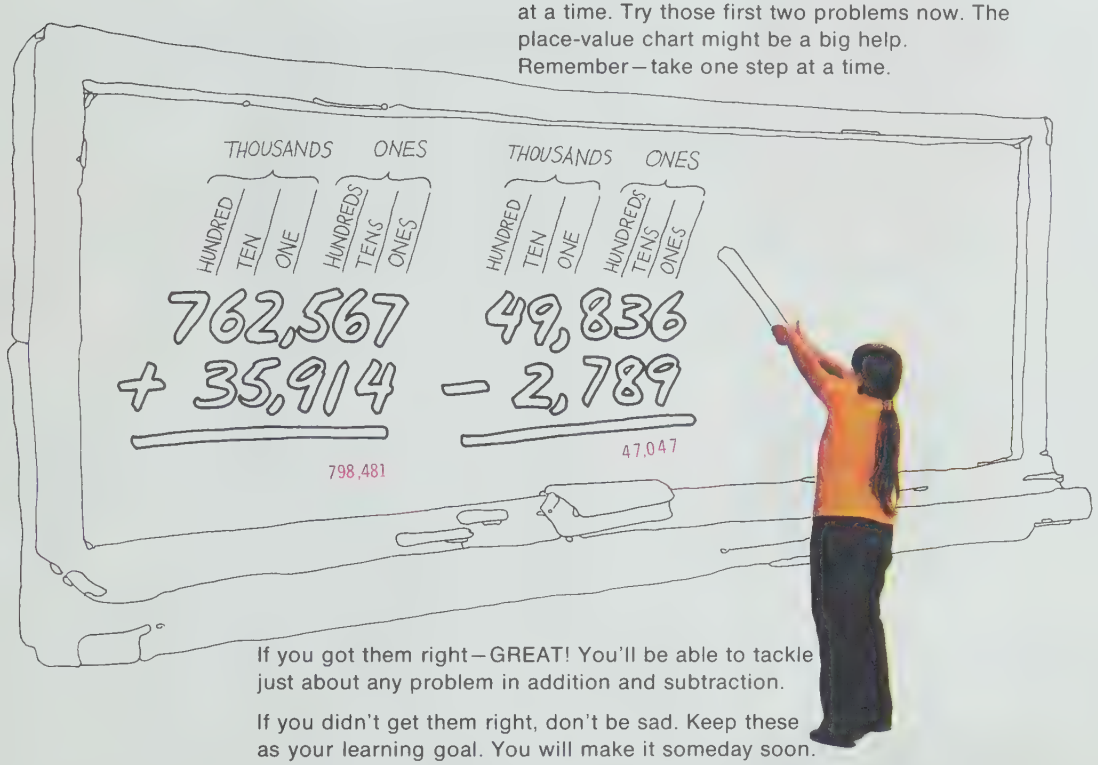
*And poor Fred! What did he do wrong in this addition?

You do these. Don't make mistakes as Fred did. Be careful. Watch the signs.

	a	b	c	d	e
9.	491 − 365 = 126	655 − 378 = 277	979 + 509 = 1488	636 + 694 = 1330	530 − 296 = 234
10.	571 − 278 = 293	927 + 385 = 1312	861 − 627 = 234	709 − 248 = 461	439 + 483 = 922

176

Remember the two problems on the first page of this chapter? They were to be your goal. You have been adding and subtracting all sizes of numbers. No matter how large the number, you took just one step at a time. Try those first two problems now. The place-value chart might be a big help. Remember—take one step at a time.



If you got them right—GREAT! You'll be able to tackle just about any problem in addition and subtraction.

If you didn't get them right, don't be sad. Keep these as your learning goal. You will make it someday soon.

177

See activity 7, page 178b.

goal Diagnosis of ability with advanced addition and subtraction skills

page 177 Youngsters who can compute these two examples accurately are now operating by concept. They should be able to compute **any** addition or subtraction problem. While praising these youngsters, don't forget to encourage those who still have not achieved this goal. They will make it!

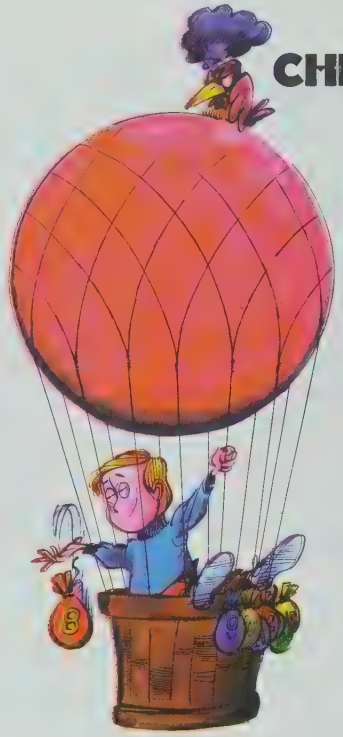
lesson Page 178

goal **Checkout**—understanding of place value; adding and subtracting two 3-digit numbers with renaming

page 178 The skills tested on this page are identified on the answer key.

If there are errors in problems 1 or 2, more work with the Magic Number Box will help. See activity 2, page 178a.

Given the extensive diagnosis of the chapter, continued errors in computation mean that the child simply is not ready to perfect these skills. Give the pupil a rest. Come back to these skills later. Chapter 15, pages 286 through 312, will again feature addition and subtraction. Please do, however, inform the child's parents of his progress and your strategy.



CHECKOUT

1. Look at the number 6425. Skill: Place value

- **a** What is the value of the digit 2? 20
- **b** What is the value of the digit 4? 400
- **c** What is the value of the digit 6? 6000
- **d** Is the 6425 the largest number that can be written with these four digits? No (6542 is.)

2. Write the largest number in each set of numbers. Skill: Place value

a	234, 432	**b**	802, 801
c	3786, 3876	**d**	300, 200, 100
e	64, 71, 59	**f**	240, 420, 402

3. Compute. Watch out! Look at the signs. Skill: Addition and subtraction

	no renaming			renaming ones			renaming ones
a	251 + 428 = 679	**b**	879 − 426 = 453	**c**	627 + 138 = 765	**d**	785 − 269 = 516
e	930 − 213 = 717	**f**	3605 + 1287 = 4892	**g**	816 − 568 = 248	**h**	1564 + 1289 = 2853

178

See activities 8 and 9, page 178b.

See activity 10, page 178b.

RESOURCES

another form of evaluation

for Progress Check–page 163
Add.

	(a)	(b)	(c)	(d)	(e)
1.	85 + 63 = 148	78 + 91 = 169	94 + 55 = 149	67 + 92 = 159	89 + 20 = 109
2.	74 + 53 = 127	96 + 32 = 128	70 + 88 = 158	84 + 54 = 138	95 + 82 = 177

for Progress Check–page 164
Subtract

	(a)	(b)	(c)	(d)	(e)
	h t o	h t o	h t o	h t o	h t o
1.	695 − 48 = 647	479 − 45 = 434	566 − 17 = 549	734 − 26 = 708	928 − 23 = 905
2.	853 − 47 = 806	282 − 55 = 227	637 − 32 = 605	886 − 36 = 850	796 − 89 = 707

for Checkout–page 178
1. Look at 8145.
   a) What is the value of the digit 4? 40
   b) What is the value of the digit 1? 100
   c) What is the value of the digit 8? 8000
   d) Is 8145 the largest number you can write with these four digits?
   No (8541 is.)
2. Ring the largest number in each set of numbers.
   a) 243, 432 b) 705, 704
   c) 4123, 4213 d) 600, 500, 400
   e) 83, 91, 79 f) 350, 530, 503
3. Compute. Watch out! Look at the signs.

a) 361 + 328 = 689	b) 798 − 537 = 261	c) 538 + 146 = 684	d) 873 − 247 = 626
e) 740 − 624 = 116	f) 4502 + 4489 = 8991	g) 628 − 389 = 239	h) 1776 + 1189 = 2965

activities

1. things spirit master or index cards

Prepare a spirit master or write the problems on index cards. Cover the cards with clear plastic or thin paper. The pupils can write on the plastic with washable crayon (it wipes off) or on the thin paper (the problems show through) so that the cards can be used again.

Name the number that is 1 less than and the number that is 1 more than each of these numbers.

____ 234 ____ ____ 900 ____
____ 300 ____ ____ 679 ____

Can you find a pattern? (The ones number goes up 1 or down 1.) Can you find more than one pattern? (One less than an even-hundreds number changes the tens and hundreds too.)

2. things index cards, small box, felt pen

Prepare a numeral card for each 1-digit number. Place them in the box—you might want to call it the Magic Number Box. Shake the box. Have a child draw 3 cards and arrange the cards to form a 3-digit numeral. *Can you form other numerals with the same digits?* Have the child read aloud each numeral he arranges.

Vary the number of cards drawn. Increase the number of possibilities by including additional sets of digit cards.

3. Use whatever technique you used for activity 1. Simply change the question and numbers as shown.

Name the number that is 10 less than and the number that is 10 more than each of these numbers.

____ 513 ____ ____ 236 ____
____ 572 ____ ____ 320 ____

Can you find a pattern? Can you find more than one pattern?

4. things set of numeral cards with 2-, 3-, and 4-digit numbers

Use this game with 2, 3, or 4 players. The cards are shuffled and dealt facedown, the same number of cards to each player. The cards remain facedown in a stack before each player.

Each player turns over the top card of his stack. The player whose card has the greatest number wins all the faceup cards. Play continues until there are no more cards.

Variations:
1. The player whose card has the least number wins all the faceup cards.
2. With only 2 players, the cards won may be placed facedown under each player's stack. Play continues until one player has all the cards.

5. Back to the same technique as for the activity on page 147, but change the questions and numbers as follows:

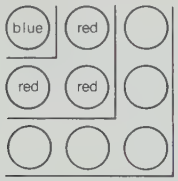
Name the number that is 100 less than and the number that is 100 more than each of these numbers.

____ 346 ____ ____ 956 ____
____ 785 ____ ____ 307 ____

Can you find a pattern? Can you find more than one pattern?

6. things crayons

Get the youngsters started making the pattern shown. Have them use a different color for each section than was used for the preceding section. Their goal is to continue until they have made a 10-by-10 array of marbles.



Now record the number of marbles in each section. Begin with the section that has only 1 marble. *What pattern do you see?* (1, 3, 5, . . . , 19) *Without adding, do you know the sum for this section?* $1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 = \square$

7. things 2 wood cubes

Write a 3-digit numeral on each face of the cubes. Pair pupils. The players alternate rolling the cubes and working with the numbers that land faceup. Points are earned as follows:

- Finding the correct sum—1 point
- Finding the correct difference—2 points

A player may earn up to 3 points per round. Players predetermine the number of points needed to win.

8. things small cards

Prepare a set of cards (30 for 2 players, 32 for 3 players, 34 for 4 players) by writing a 2-, 3-, or 4-digit numeral on each card.

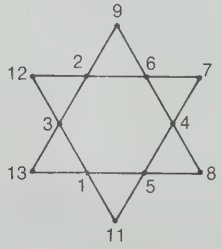
The dealer puts 2 cards faceup in the center of the table and deals the remaining cards equally to all players. The first player examines his cards, looking for a number between the two numbers shown by the cards on the table. If he does not have one, he passes. If he has such a card, he places it on the table, takes the original 2 cards and places them facedown in front of him. Then he lays down another card faceup from his hand to form a new pair of numbers for the next player. Play continues until one player has used all his cards or until everyone must pass. The winner is the player who has the most facedown cards in front of him.

9. things numeral cards 0 through 9; spirit master place-value chart (see page 72a); box

Provide each pupil with a place-value chart. Mix the cards in a box. Predetermine whether everyone will form a 1-, 2-, 3-, or 4-digit numeral. One pupil draws a card and reads it to the group. Pupils write this digit in any one of the place-value positions on their charts. Repeat until the appropriate number of digits has been drawn. The winner is the one who makes the largest (smallest) number. Ties are possible.

Variation: Replace each card after the numeral has been read.

10. things spirit master as shown



1. Add the numbers at the corners of each small triangle. What did you find? (Each sum is 17.)

2. Add the numbers at the corners of each large triangle. Anything special? (Both sums are 30.)

3. Add the 4 numbers along each side of the large triangles. What did you find? (Each sum is 27.)

additional learning aids

notation—chapter objectives 1, 2

SRA products

diagnosis: an instructional aid—Mathematics Level A, SRA (1973)
Probe: L-7
Skill through Patterns, level 3, SRA (1974)
Spirit masters: 52
Visual Approach to Mathematics, level 3, SRA (1967)
Visual: 12

other learning aids (described on page 216g)—Abacus board, Abacus Spinner Game, Chip Trading, Place Value I and II

operation—chapter objectives 3, 4, 5, 6

SRA products

Computapes, SRA (1972)
Module 2, Lessons: AS 25, 27, 36
Cross-Number Puzzles (Whole Numbers), SRA (1966)
Addition card: 13
Subtraction cards: 7, 8, 9, 10, 11, 12, 13
Mathematics Involvement Program, SRA (1971)
Card: 26
Skill through Patterns, level 3, SRA (1974)
Spirit masters: 48, 49, 50, 51, 53

other learning aids—Veri-Tech Senior (addition and subtraction books)

9 MEASUREMENT LENGTH

before this chapter the learner has—

Mastered measuring lengths to the nearest centimetre or inch

in chapter 9 the learner is—

1. Selecting an appropriate metric or customary unit to measure a length
2. Measuring lengths from 0 to 10 metres and from 0 to 30 feet with appropriate units of measure

in later chapters the learner will—

1. Master selecting appropriate metric or customary units to measure a length
2. Master measuring lengths from 0 to 10 metres and from 0 to 30 feet with appropriate units of measure

Notes & Things

This chapter on measurement is a "doing" chapter. Each page requires action and thought but not always written answers to questions. The amount of time you will need to complete these pages will depend on the previous measurement experiences your pupils have had. Your function will be to guide each child's learning experiences—encouraging each one to go as far as possible, helping the individuals who hit a trouble spot, and making sure that everyone becomes actively involved in the process of measuring real things.

If your pupils work well in small groups and if you have some independent readers, you may want the groups to conduct themselves as young scientists. You can then serve as chairman for the progress reports of the scientific investigators. After the groups perform their lab exercises and complete the questions in their books, you will want to call them together and have each group report its discoveries. It is hoped that each group will be finding applications of measurement on its own.

You will find equal emphasis on the metric system and the customary system of measure. The pupils will not be asked to convert a measurement from the customary system to the metric system, but they will be asked whether 1 inch is longer or shorter than 1 centimetre, for example.

You deserve an explanation of the total program's approach to metric measure. First of all you will see the metric system related to the real world. Children will use the most common units of metric measure in problem-solving situations. The patterned prefixes will be used in concept development sequences starting at level 6; they will also be used as an application of our numeration system.

It appears that the importance of world trade is one of the main reasons that motivates a nonmetric country to switch to the metric system. Since all types of international communication are becoming more and more important, we decided to go international all the way in this program. The units of measure and their spelling will follow the recommendations of the international forum called the General Conference of Weights and Measures. They have recommended the Standard International System. It has basic units for length (metre), mass (gram), time (same old units), temperature (Celsius or centigrade), electric current (same units again), and illumination (no change here either). Considering that there are six fundamental measurement units in the Standard International System and that we already know three of them, maybe things won't be so bad after all. It's O.K. if you or your school prefers to use another spelling of the metric units of measure. In either case, the children should be aware of the two spellings of many of the words.

Please note that no abbreviations are used on the pupil pages in this chapter. Use them if you think it is appropriate. This program will *not* use a period after the metric units cm, m, and the like. These are considered symbols rather than abbreviations.

things

rulers marked with centimetres and inches
pieces of string
chalk
strips of paper
yardsticks and metresticks
yarn
adding-machine tape

For the extra activities you will want to have these things available:
- strips of cardboard, floor tile, or coated wire
- pictures of Sears Tower (Chicago) and Empire State Building (New York City)

goal Think about and explore ideas through a picture clue

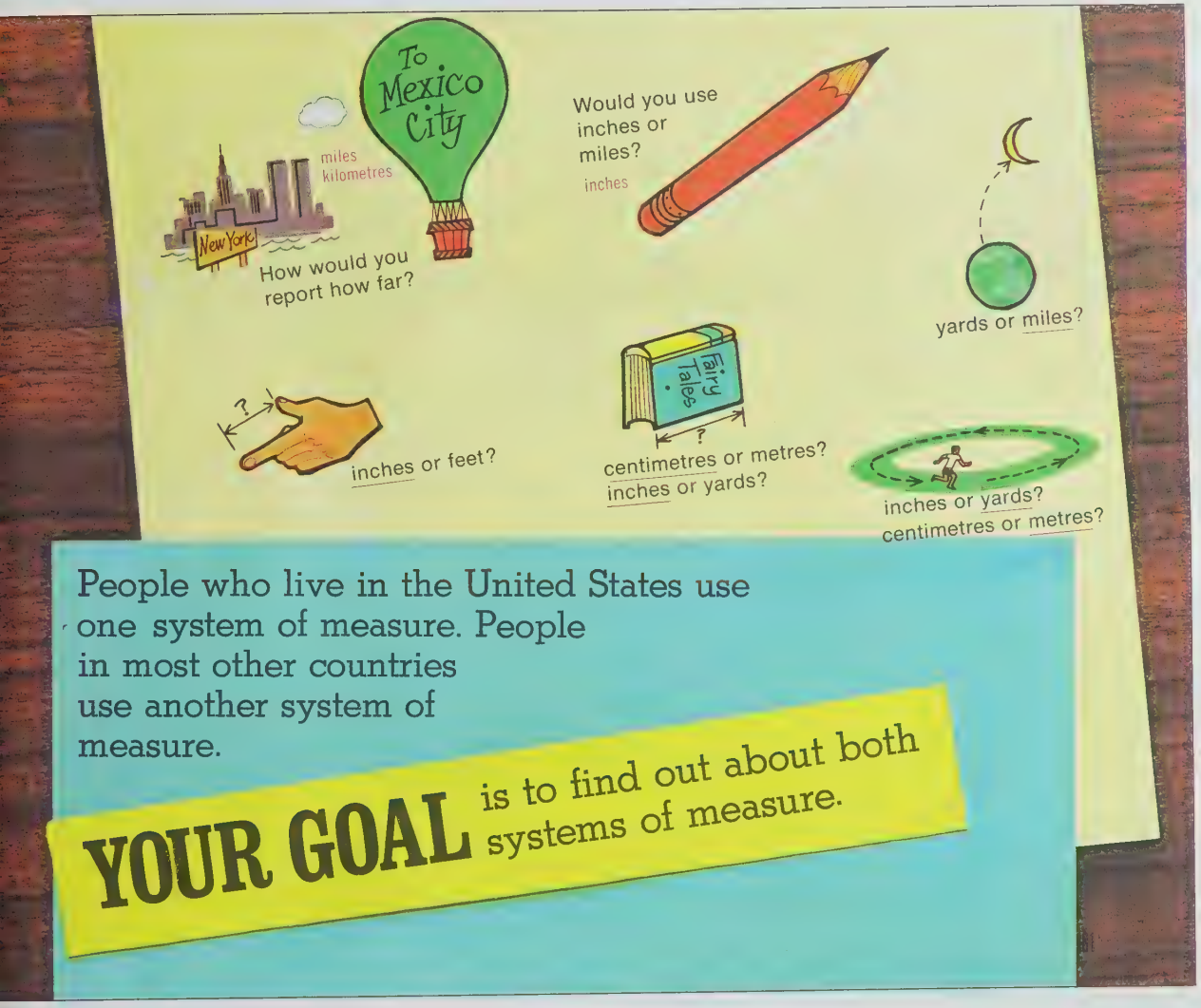
page 179 Only one question is needed to get the discussion going for this page. The children in the picture are measuring the length of a dog. This measurement probably isn't very important, but sometimes a measurement is. **When?** Can the pupils remember when the idea of length has been important in their lines?

You should get a wild variety of answers. Some of them may relate to things the child has purchased, such as kit string, rope, or ribbon. Some may think of lengths associated with sports, such as the length of a football field, a race track, and so on. The answers may seem unrelated if you have some good independent thinkers. If the answers sound like a broken record, help the children think of lengths associated with height, width, or distance.

lesson Pages 180, 181, 182

goal **Survey**—using appropriate units of measure

page 180 **Think** *and* **Talk**



People who live in the United States use one system of measure. People in most other countries use another system of measure.

YOUR GOAL is to find out about both systems of measure.

goal **Survey**—using appropriate units of measure

warm-up Start out with some awareness questions. Have fun. Get everyone thinking about measuring. Ask questions such as these. *Is your arm longer than your leg? Is your hand longer than your foot? Can you put the fingers of one hand around your other wrist? around one ankle?*

Can you make the fingers on both your hands touch if you put them around your waist? around your head? around your neck? around your knee? Are your hands the same length? the same width?

Are you taller than any chair in this room? Are you taller than the table is long? How many of you high is the door?

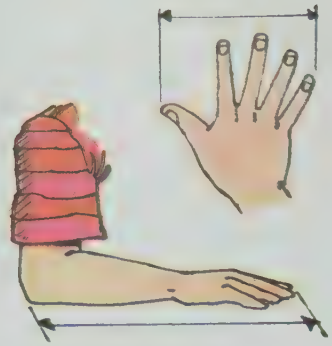
page 181 Who knows what about units of measure? Share ideas and find out. Let pupils jot down their choices of units of measure for the examples on the page. Talk about their choices. This will give you clues to their previous experience with measuring.

Does anyone know any other names for measurement? If pounds are mentioned, say that the pound is a unit of measure for weight. Cups? That is a unit of measure for liquid. This type of discussion will help you broaden the goal. We measure so many things. For now the goal is to learn as much as possible about the measurement of length (distance).

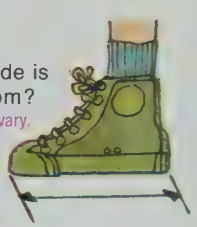
goal Exploring arbitrary units of measure

page 182 Strictly for exploration and sharing ideas. Each child needs to know that if his handspan is 14 centimetres, for example, he has a handy built-in ruler for estimating other measurements.

Each child needs to be able to visualize his height and use it in estimating other heights. He should be able to hold two fingers apart and be confident that he can estimate a centimetre or an inch. He should be able to hold his hands apart and estimate 10 centimetres. But this internalization of measurement comes only with practice—lots of it. Your asking about how long, how wide, how far things in the classroom are will help every child develop a sense of measurement.

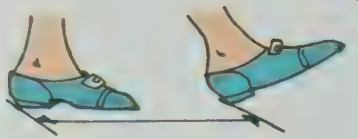


1. How many wide is your open book?
Answers will vary.



2. How many wide is your classroom?
Answers will vary.

3. How many wide is the chalkboard?
Answers will vary.



4. How many long is the hallway?
Answers will vary.

This is how people measured things long ago.

5. Use your hands as a unit of measure.
   - a What is the width of your desk? Answers will vary.
   - b Are your friends' desks the same size? Yes
     What measurements do your friends get?
     Were they the same? They may be.
   - c Why are the measurements different? Everyone has different-size hands. (People may have different-size desks, too.)

6. Is the width of every person's hand always the same? No

Compare Find out.

Would a measuring stick marked with the same-size units help? Yes
Would the measurements of the desks be the same if you used the same measuring stick? Yes—provided that all desks are the same size

182

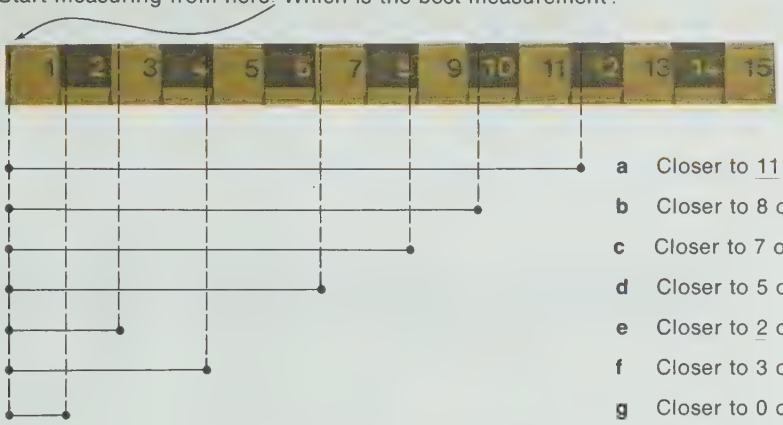
things strips of cardboard, floor tile, coated wire, or anything that is rigid

Cut the strips to measure 1 cm, 1 inch, 10 cm, and 12 inches. Ask a pupil to close his eyes, pick a strip, and guess how long it is.

Using something flexible, such as string, is too difficult. If you don't believe it, try it yourself.

Some measuring sticks are called rulers.
One is drawn on this page.
It is marked with a unit of measure called a centimetre.

1. Start measuring from here. Which is the best measurement?



a Closer to 11 or 12?
b Closer to 8 or 9 or 10?
c Closer to 7 or 8?
d Closer to 5 or 6?
e Closer to 2 or 3?
f Closer to 3 or 4 or 5?
g Closer to 0 or 1 or 2?

2. Could you use a ruler like the one above to measure your height? Yes

3. If you wanted a longer ruler, could you make a centimetre ruler 30 centimetres long? Yes
Could you make one 50 centimetres long? Yes
Could you make one 100 centimetres long? Yes

4. A centimetre is called a standard unit of length.
What could the word *standard* mean? Always the same

5. What things might you measure using a centimetre ruler?
Discuss. Smaller things such as books, paper, or stamps would be most practical.

183

lesson Pages 183, 184, 185, 186, 187

goal Experience in measuring to the nearest centimetre

memo Please don't be concerned because the pupil may have had little or no experience with a unit of measure. The centimetre, for example, is a standard unit of distance just as the numeral 5 is a standard symbol for a number. The children have learned "numberness." They will also learn measurement if they have activities and practice comparable to those they had with numbers.

page 183 Strictly for discussion. Please don't ask for recorded answers. Give pupils an oral choice of the two measurements for each of the parts of problem 1 so that they won't be distracted from the line segment itself. They will be busy following that dotted line—a big job all by itself.

things stiff paper for making rulers

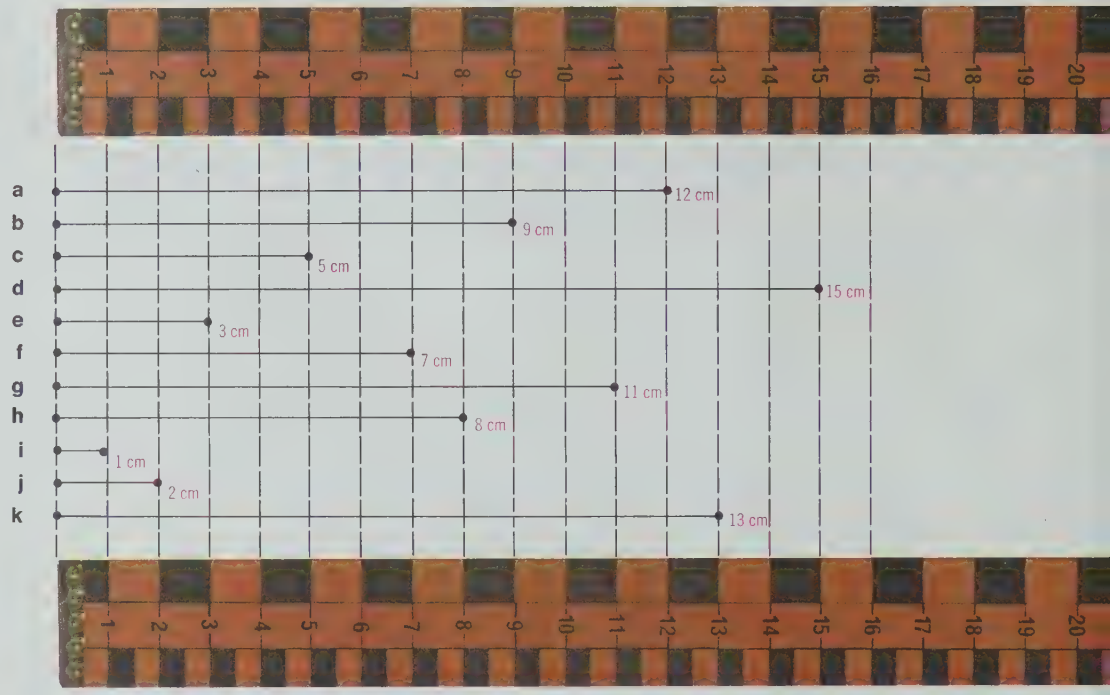
Exercise 3 signals an extension activity. Let the pupils who are interested make the rulers suggested. Then have them name an object that they think is about the same length as each ruler. Have them measure to find out if their estimates are accurate.

goal Practice in measuring with a centimetre ruler

page 184 Please notice that picturing two rulers enables the child to align a sheet of paper with the unit above and that below. This paper mask will serve to focus attention on the length of the line segment rather than on the length of the ruler.

Many people use cm to stand for the word centimetre.
You will see both used on the next pages.

Centimetres are pictured on the two rulers below.



1. What is the measurement of each segment shown between the two rulers?

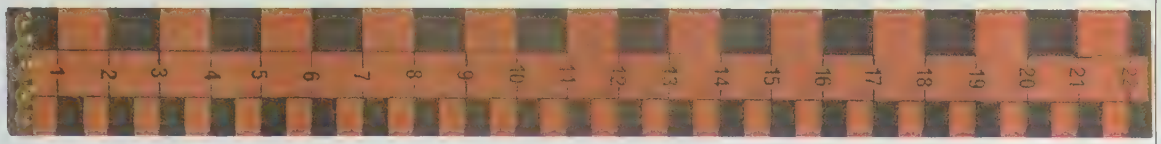
184

You might play an "I Spy" game to fill up some spare minutes waiting for a bell to ring.

I spy something in the front of the room that is longer than 1 inch but shorter than 1 foot. What is it?

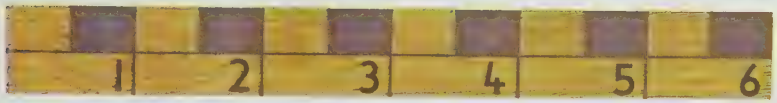
I spy something on the bookcase that is longer than 5 cm but shorter than 20 cm. What is it?

1. Look at the segment marked above the ruler.
Is it closer to 9 or closer to 10 centimetres long?



2. Can you find any rulers that do not show centimetres? Yes

3. One kind of ruler is marked with inches. The length of 1 inch is another standard measuring unit.



How many inches are marked on this ruler? 6

a Which is more—2 inches or 3 inches? **b** Which is more—6 inches or 4 inches?

4. Could you make a ruler that is 12 inches long? Yes
Could you make one that is 36 inches long? 60 inches long? Yes
Yes

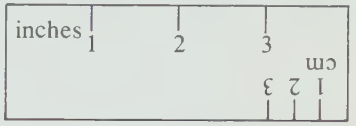
5. Measure the length of your finger with either a centimetre or an inch ruler.
How many centimetres? How many inches? Accept reasonable answers. Range: Between 5 and 8 cm and between 2 and 3 in.
Is the number of centimetres the same as the number of inches? No

6. Pretend you measured the length of string with an inch ruler.
I measured another piece with a centimetre ruler.
Could we add our measurements together and know how much string we had in all? No. The units of measure are different.

goal Exploring centimetres and inches as units of measure

things rulers marked with centimetres, inches
pieces of string
chalk
strips of paper

memo Store-bought rulers can look so different from one another. Most of them will have inches marked on one side and centimetres on the other. But look out! Where is the zero marked, if at all?



If there is no zero or if the ruler has to be rotated to use another scale, you have to take time out for a lot of practice in just handling the ruler.

page 185 Get acquainted with the rigged rulers on the page before you have the pupils work with real ones. Have pieces of string, chalk, strips of construction paper, and paper clips handy for more measuring experience. Any one of these items can be put on the pictured rulers and measured. Use one scale; then use the other. This is the first chance you have to establish that the centimetre is one unit of length; the inch is another. It's O.K. to use either unit of measure.

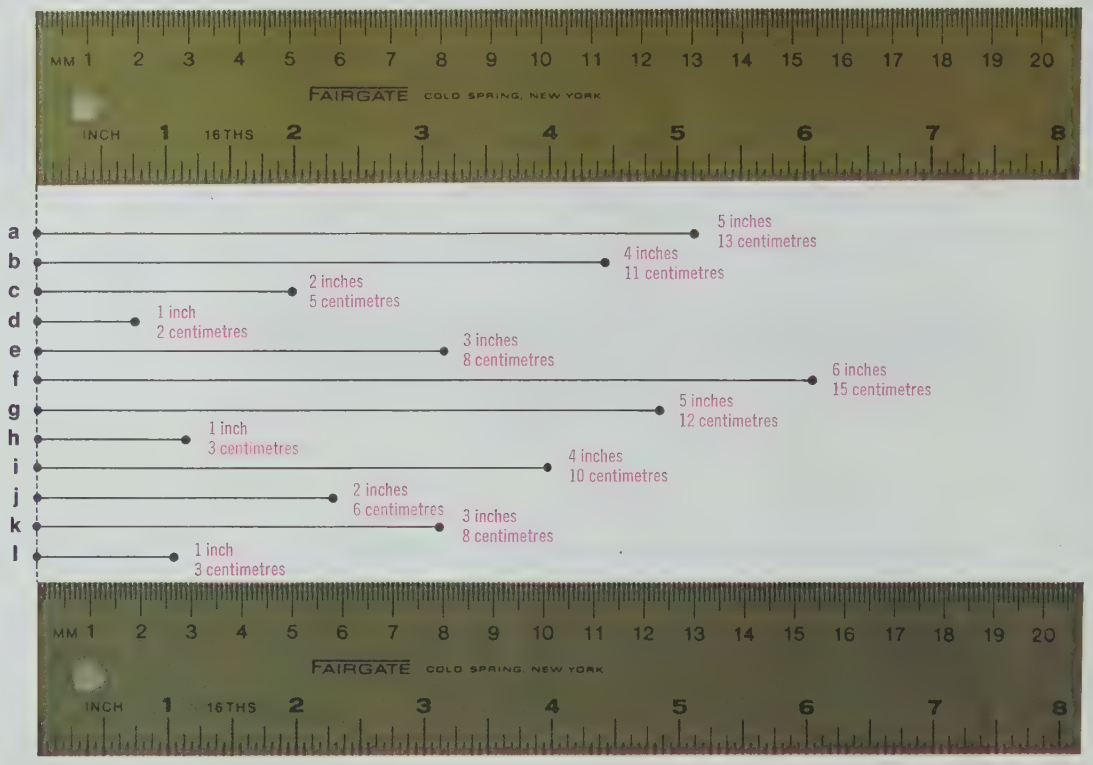
goal Practice in measuring line segments

memo Look out for trouble. The rulers pictured on the page are marked with centimetres and inches, but this marking may look different from the real rulers the children have. Both scales have zero marked, and both start from the left. Each of the two sets of numerals is a different size, but that still won't prevent confusion. Take time to study the pictures on the page before you begin. Note the contrast between the real rulers and the pictures after the page activities rather than before.

page 186 Use of the paper mask to align the two ruler scales and the length of a given segment will help. There will be less confusion if **all** the segments are measured to the nearest inch before they are measured to the nearest centimetre.

These rulers have both centimetres and inches marked.

1. Each segment below is nearer to what number of inches?



2. Go back and measure each segment to the nearest centimetre.

186

1. Measure to the nearest inch.

a ●——————————● 4 inches

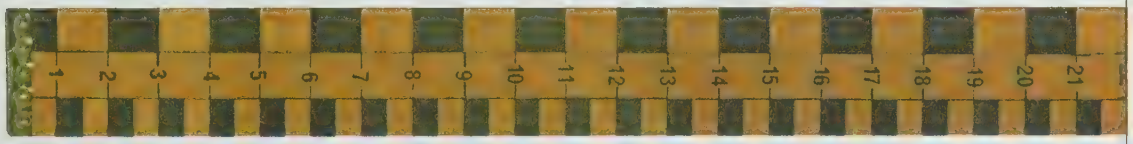
b ●——————————● 3 inches

Is the length of the first segment 6 inches? Why? No It starts at 2 on the ruler, not at 0.
Is the next segment 5 inches long? Why? No it starts at 2 on the ruler, not at 0.

2. Measure to the nearest centimetre.

a ●——————————● 6 centimetres

b ●——————————● 9 centimetres



Is the length of the segment in **2a** 11 centimetres long? Why? No. It starts at 5 on the ruler, not at 0.
Is the length of the segment in **2b** 14 centimetres long? Why? No. It starts at 5 on the ruler, not at 0.

3. Think carefully. Do you have to start measuring from 0? No

4. Do you suppose there is a special name for a ruler with 100 centimetres marked on it? Find out the name of that ruler. Metrestick

187

goal Reinforcing the concept that measurement begins at zero

memo This is not a trick page. The idea that measurement begins at zero is too frequently ignored in a child's learning experience. This page hits the problem head-on. A great deal of practice may be needed with the ideas on this page.

page 187 The children's experience with the number line helps get across the message that we **start** with zero when measuring, **or** we **count** individual units. A knowing youngster who has helped a do-it-yourself parent might use subtraction to find the measure, but don't count on this or try to teach it now.

lesson Pages 188, 189

goal Introduction to the yardstick and metrestick

things yardsticks and metresticks

warm-up You'll want to provide hands-on experience with the yardstick and metrestick before discussing the page. Try measuring the width of the door, the room, and the chalkboard with each type of ruler. Record these measurements in yards and in metres.

page 188 Notice that comparisons are made between customary and metric measures, but no conversions are required. The youngsters are asked only which measure is longer. Let's keep it that way – O.K.?

Don't tell the answers to problem 3. Put your researchers to work.

188

Length Measures	
Some countries of North America	Most other countries
inch	centimetre
yard (36 inches)	metre (100 centimetres)

1. Find a yardstick and a metrestick.

   a Compare the length of 1 inch and 1 centimetre. Which is longer? 1 inch

   b Compare the lengths of 1 yard and 1 metre. Which is longer? 1 metre

2. RULERS COME IN ALL SIZES
   Some have 12 inches marked on an edge.
   Some have about 30 centimetres on an edge.
   Some are longer –
   One with 100 centimetres marked on it measures 1 metre. It's a metrestick.
   One with 36 inches marked on it measures 1 yard. What's it called? Yardstick

3. Think about these. Try to find an answer.

   a How is cloth sold? By the yard

   b How long is a football field? 100 yards

   c When you buy ribbon, how is it measured?
   By the yard (or fraction of a yard) Prepackaged ribbon comes in feet or inches.

   d How is your height measured? feet and inches
   Why isn't the unit yards used? It's too big a unit.
   Could a person be 2 yards tall? Yes
   Could a person be 3 yards tall? Yes, but it's not likely.

1. Alice measured the length of the classroom in metres.
Annie measured the length in centimetres.
They got different numbers. Why? They used different-size units.

2. Dave measured the length of the classroom in metres.
Don measured the length of the classroom in yards.
They got different numbers.
How do you explain the difference? They used different-size units.

3. Complete the tables.

a

centimetres	metres
100	1
200	? 2
300	? 3
400	? 4
600	? 6
800	? 8
1000	? 10

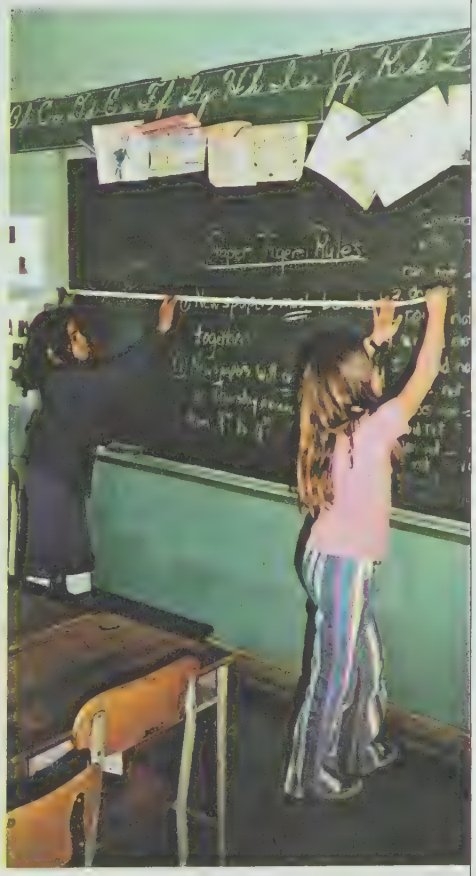
b

centimetres	metres
100	1
400	? 4
600	? 6
800	? 8
1000	? 10
300	? 3
200	? 2

4. Do you have a ruler in your room that measures 12 inches?
Maybe someday this kind of ruler will not be used anymore.
It is used now though, isn't it?
We should know what the special name for 12 inches is.
The distance measured by 12 inches can be called a foot.
Where do you suppose that name could have come from?
A man's foot is about 12 inches long

a Is a length of 12 inches longer than a length of 12 centimetres? Yes

b Is a length of 12 inches longer than a length of 1 metre? No



189

goal Practice in finding related measurements (metres, centimetres); introduction to foot

page 189 There is lots of work on this page. Actually doing the measuring tasks will clear up any confusion.

Who uses multiplication rather than repeated addition and who uses division rather than repeated subtraction to find the answers for problem 3? You can find clues to your pupils' abilities.

The history of measurement contains some interesting stories about the origin of the name **foot** as a unit of measure. Finding the origin of various units of measure makes a good research project for independent learners.

Posters depicting a unit of measure as well as its origin are commercially available. Why not let the pupils make their own posters for the room?

lesson Pages 190, 191, 192

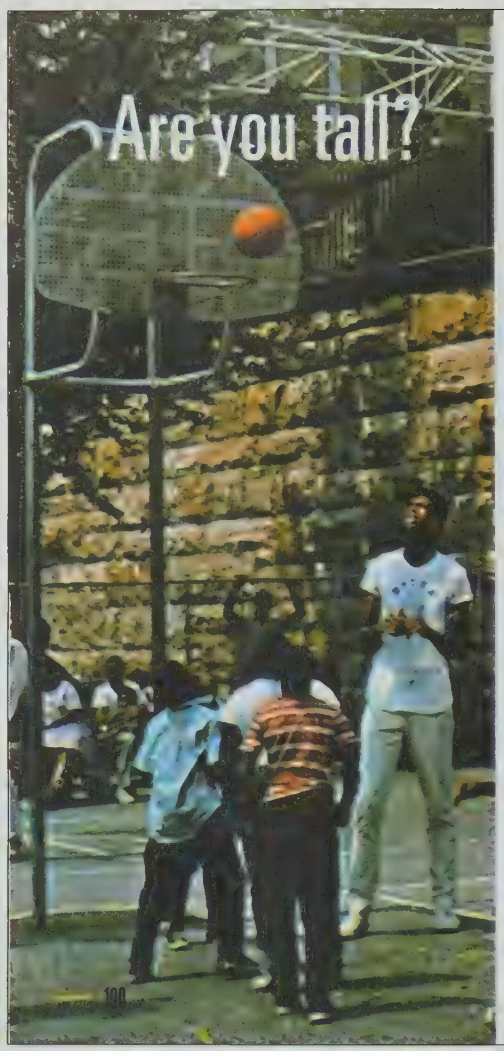
goal Finding related measurements (inches, feet, yards); naming appropriate units of measure

memo It would help if each child cut a strip of adding machine tape or a piece of yarn that is as long as he is tall. If these lengths are then put up on a wall, with one end touching the floor, the child could see his height again and again. Be sure everyone puts his name with his height.

If the child knows the measurement of his height (length), he will have made a big step forward in acquiring one of the concepts of measurement – that there is a given length that corresponds to his given height. Though he is not in the same place as the yarn, his length is still represented by that yarn.

things yarn
adding machine tape

page 190 There are some wild questions on this page. Allow time for thinking and exploring. Let answers serve as clues to types of additional measuring experiences needed.



Think about a basketball team for a minute.
Are the players tall? Yes, most are.
Are you taller than a basketball player? No
Could a basketball player be 6 feet tall? Yes

The standard unit called a foot is longer than an inch BUT shorter than a yard.

12 inches measure 1 foot of length
3 feet measure 1 yard of length

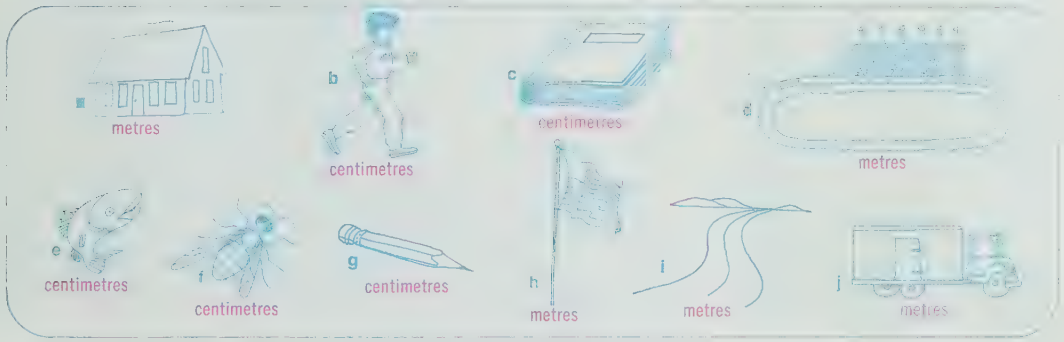
1. 12 inches measure 1 foot.
   Does 1 foot measure the same as 12 inches? Yes
2. 3 feet measure 1 yard.
   Does 1 yard measure the same as 3 feet? Yes
3. You may want to draw a picture to help with this question. If 1 foot measures 12 inches, how many inches measure the same as 1 yard? 36 inches
4. Can you find a ruler that measures 1 foot?
   What other measure is marked on the ruler? inches (sometimes centimetres)
5. Make a list of those things you know that would measure less than 1 foot. Answers will vary.
   Examples: your finger, a hotdog, a pen, an envelope
6. Make a list of things that measure less than 1 yard. Answers will vary.
   Examples: your arm, your desk top, your book or notebook, a candle
7. What things measure about 12 inches, or 1 foot, long?
   - **a** Is this sheet of paper about 12 inches, or 1 foot, long? Close to it
   - **b** Is your shoe about 12 inches, or 1 foot, long? It could be.
   - **c** Measure a man's shoe.
     Is that length about 12 inches, or 1 foot? Probably
   - **d** Measure a woman's shoe.
     Is that length about 12 inches, or 1 foot? Probably

things adding-machine tape or yarn

Cut 2 same-length pieces of tape or yarn. Repeat for 5 different lengths. It makes no difference how long the strips actually are, but make sure that there is a difference of at least 5 cm or 2 inches between pairs of lengths. At random around the room display 1 of the 2 same-length pieces in a horizontal position; put the other in a vertical position.

Challenge a child to find 2 pieces that are the same length without moving the pieces and without measuring them.

1. Is there any difference between "how long" and "how tall"? Discuss. Suggestions are given.
   - **a** How many inches long are you? } Should be the same answer
   - **b** How many inches tall are you? }
   - **c** How many centimetres long are you? } Should be the same answer
   - **d** How many centimetres tall are you? }
2. How many centimetres measure the same distance as a metre? 100
3. How could you measure the length of your classroom?
   - **a** Step it off. How many steps is it? Answers will vary
   - **b** Is one of your steps a standard unit of length? Probably not
   - **c** Get a metrestick. How many metres is it? Answers will vary with classrooms.
4. You are going to tell someone about the length of a room. Which measurement could you use so that they would understand? feet, yards, metres
   - **a** Could you report the length in centimetres? Yes
   - **b** Would there be more centimetres than metres? Yes
5. Would you use centimetres or metres to measure the real things pictured below?



191

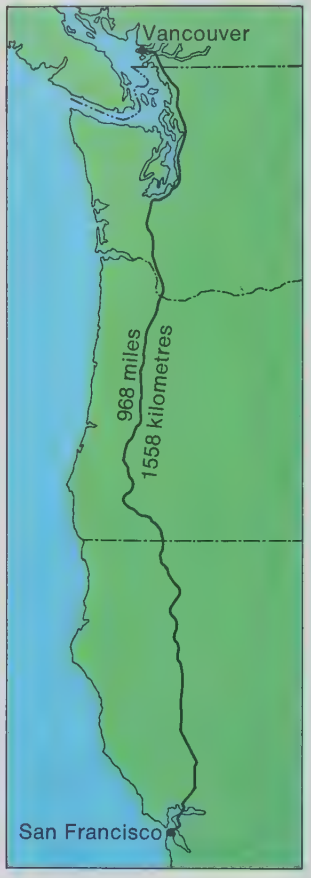
goal Practice in selecting an appropriate unit of measure

page 191 If pictures and words are too abstract, close the book and provide similar experiences with real objects in the room. Once again, let your youngsters' answers serve as clues to additional experiences needed.

goal Introduction to miles and kilometres as units of measure for long distances

memo Remember that very few adults really have an image of the length of a mile or a kilometre. We adults generally think of 1 mile, for example, as the distance an expert runner can cover in 3 to 5 minutes, or as the distance of about 8 city blocks. We certainly would have trouble visualizing 176 football fields placed end to end. The length of a kilometre is only about two-thirds of a mile, but that's still too long a length to visualize. (Some adults have trouble estimating the length of 1 metre or 1 yard, for goodness' sake!) Be patient with the children as they work on this page.

page 192 Help! There are a lot of words. These ideas are not to be mastered; rather, they attempt to relate some very abstract ideas to the real world. Talk together and share ideas.



Would you measure the distance from San Francisco to Vancouver (Canada) in feet? in yards? in miles?
No No Yes

Many people living in North America measure long distances in miles. A mile is a standard unit of measure.

5280 feet measure the same distance as 1 mile.
1780 yards measure the same distance too.

Answer these questions.

1. Dan's father drove 38 miles the first hour. He drove 53 miles the second hour. How many more miles did he drive the second hour? 15

2. A carrier pigeon flies 29 miles on one trip. It flies 45 miles the next trip. How far did the pigeon fly on both trips together? 74

Many people measure long distances in kilometres. A kilometre is also a standard unit of measure.
1000 metres measure the same distance as 1 kilometre.

3. Would a person living in Europe measure the distance from Paris to Rome in centimetres? in metres? in kilometres? Long distances in Europe are usually measured in kilometres. No No Yes

The length of 1 kilometre is less than the length of 1 mile.

4. The measurement system that uses centimetre, metre, and kilometre is called the metric system. Many people think that the metric system should be used all over the world. Can you think of a reason why? Discuss. Examples: Everyone would be using the same units of measure. It's easier for trade, manufacturing, communication.

192

things · pictures of Sears Tower (Chicago) and Empire State Building (New York City), graph paper, paste

Explore the possibilities for picturing 1 mile. The Sears Tower is 1450 feet tall. *About how many Sears Towers high is a mile?* (4) The Empire State Building is 1250 feet high. *How many of this building would it take to show a mile?* (More than $4\frac{1}{2}$)

Now let's relate to something closer to home—a football field. Pretend 1 unit square of graph paper is as long as a football field. *How many squares of graph paper would show a mile?* (176! That's a big job!)

Length Measures	
Some countries of North America	Most countries
inch	centimetre
foot (12 inches)	
yard (36 inches)	metre (100 centimetres)
mile (1760 yards)	kilometre (1000 metres)

1. Complete the tables.

a

yards	feet	inches
2	? 6	? 72
? 3	9	? 108
4	? 12	? 144
5	? 15	? 180

b

metres	centimetres
1	? 100
2	? 200
3	? 300
? 4	400

2. Pretend you are in Europe. Try to find what unit of measure you would use to answer my questions.

   a How wide is the soccer field? metres
   b How far is it to the next town? kilometres
   c How big is the book? centimetres
   d How tall is the building? metres
   e How long is the swimming pool? metres
   f How high is the ski jump? metres
   g How wide is the ribbon? centimetres
   h How long is the new baby? centimetres
   i How long until dinner? (Look out!) hours and/or minutes
   j How long is the TV program? (There's another one!)

3. Do you suppose there is any difference in the way people all over the world talk about time?

The only difference is the language. The units of time are still the same. (Once in a while you will run into time situations where a 24-hour day is used rather than two 12-hour periods identified as A.M. and P.M.)

193

See activity 1, page 196a.

See activity 2, page 196a.

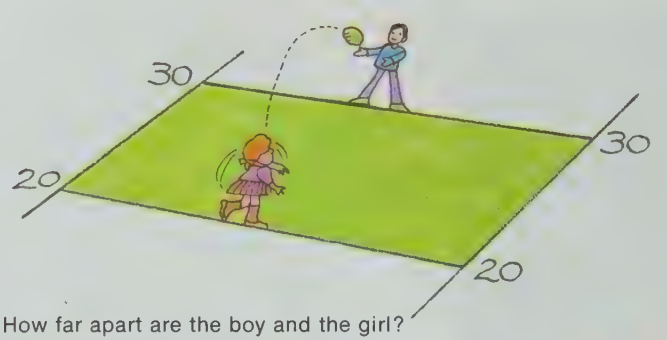
lesson Pages 193, 194

goal · Practice with related measurements

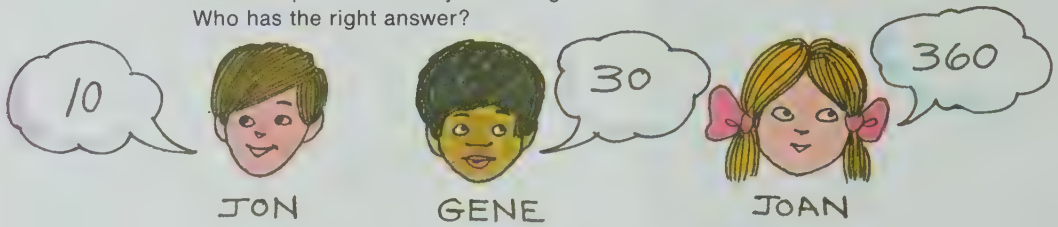
page 193 This page requires some research. Not all pupils will be able to handle this work. Consider dividing the tasks up among small groups or individual pupils. Provide time to report research findings on a later day.

goal Exploring ways to communicate measurements

page 194 A number without a unit of measure is meaningless. Use the idea to challenge pupils to report the length of the room, giving the number only. Let the others guess the unit of measure that is needed before the number makes sense. Then tackle the page.



1. How far apart are the boy and the girl?
Who has the right answer?



Jon is right if he means 10 yards.
Gene is right if he means 30 feet.
Joan is right if she means 360 inches.
Are 10 yards, 30 feet, and 360 inches all the same measure? Yes

2. Could Jon, Gene, and Joan be talking about distances they would use in Europe? Could all of them be right if they were using centimetres or metres? No No

3. Does a number all by itself mean anything when you are talking about a measurement? If someone said the distance was 100, what else would you need to know before you could understand the distance? No The unit of measure

194

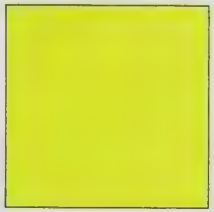
Every square is a rectangle, but every rectangle is not a square.

1. What makes a rectangle a rectangle? Look at the corners and sides of each of these rectangles. Try to get a good answer.

A rectangle has 4 square corners and 4 straight sides.

a 

b 

c 

d 

e 

2. Try measuring the lengths of opposite sides. What can you say about those measurements?

They are equal.

3. What is the distance around each of the rectangles above?

a 12 cm b 16 cm c 16 cm d 20 cm e 24 cm

lesson Page 195

goal Measuring the length of the sides of rectangles

things centimetre rulers

page 195 To test whether a figure is a rectangle, we look for–

- 4 square corners
- Opposite sides of equal length

After completing the lesson, examine the square. Does a square have 4 square corners and opposite sides of equal length? A square has one more special feature–all sides have the same measurement. Find some squares to measure. Does anyone think to measure only one side and then multiply by 4? If so, give oodles of praise. That is really **good thinking.**

lesson Page 196

goal **Checkout**—identifying the greater of two measurements

page 196 The appropriate generalizations about the two systems of measure are presented on this page.

This is demanding work. It may take more than a day to complete it. Consider some more measuring activities that can be done in the classroom if relief is needed.

CHECKOUT

1. Which is longer? Skill: Comparing units of measure

a	1 inch 1 foot	**b**	1 foot 1 yard	**c**	1 yard 1 mile
d	1 metre 1 kilometre	**e**	100 metres 1 kilometre	**f**	1000 centimetres 1 metre
g	3 feet 3 yards	**h**	10 feet 3 yards	**i**	30 inches 3 feet
j	40 feet 15 yards	**k**	3 inches 3 centimetres	**l**	1 yard 1 metre
m	1 mile 1 kilometre	**n**	1 metre 3 feet	**o**	1 centimetre 1 inch

2. Which is longest? Skill: Comparing units of measure

a	1 inch 1 foot 1 yard	**b**	12 inches 12 feet 12 yards	**c**	36 inches 36 feet 36 yards
d	36 inches 3 feet 1 yard All are the same.	**e**	1 centimetre 1 metre 1 kilometre	**f**	100 centimetres 100 metres 100 kilometres
g	100 centimetres 10 metres 1 kilometre	**h**	1 kilometre 1000 metres 10 miles		

196

things adding-machine tape

Cut lengths of tape. Use measurements on the page as a guideline. Pupils should measure and lable each length. Put lengths of tape in sets of 2. Tell which measurement is longer (shorter). Then make sets of 3 lengths. Tell which is the longest (shortest).

things checkerboard

Puzzler: How many squares can you find on a checkerboard? The right answer is not 64. (Solution: 204 squares can be found.) How many different-size squares can you find? (8)

RESOURCES

another form of evaluation

for Checkout—page 196

1. Which is longer?

a) 1 inch / 1 foot

b) 1 yard / 1 foot

c) 1 mile / 1 yard

d) 1 metre / 1 kilometre

e) 1 kilometre / 100 metres

f) 200 centimetres / 1 metre

g) 4 feet / 4 yards

h) 7 feet / 2 yards

i) 36 inches / 2 yards

j) 20 feet / 5 yards

k) 2 inches / 2 centimetres

l) 1 metre / 1 yard

m) 1 kilometre / 1 mile

n) 36 inches / 1 metre

o) 1 inch / 1 centimetre

2. Which is longest?

a) 1 inch / 1 foot / 1 yard

b) 12 feet / 12 yards / 12 inches

c) 36 yards / 36 inches / 36 feet

d) 36 inches / 1 yard / 3 feet — All are the same

e) 1 metre / 1 centimetre / 1 kilometre

f) 100 centimetres / 100 metres / 100 kilometres

g) 200 centimetres / 20 metres / 2 kilometres

h) 10 miles / 1 kilometre / 1000 metres

activities

1. things adding-machine tape

Cut and label the following lengths of adding-machine tape:

- 1 metre—with centimetres marked
- 1 decimetre—with centimetres marked
- 1 foot—with inches marked
- 1 yard—with feet marked

Provide more measuring experiences. Have the youngster use his lengths of tape to measure the following:

- His desk
- A table
- The width of the door
- The length of the room
- The height of your desk

For each example, have him decide which lengths were good ones to use and which were not good.

2. things metric and customary rulers; metrestick; yardstick

Pair pupils. Have each pair choose six distances in the room (or school) to measure. Next have them list their choices on a chart and estimate what they think each distance is. Then have them check the closeness of their estimates by measuring each distance and recording the results. Talk about how an appropriate unit of length was selected and the accuracy of their estimates.

additional learning aids

concept—chapter objectives 1, 2

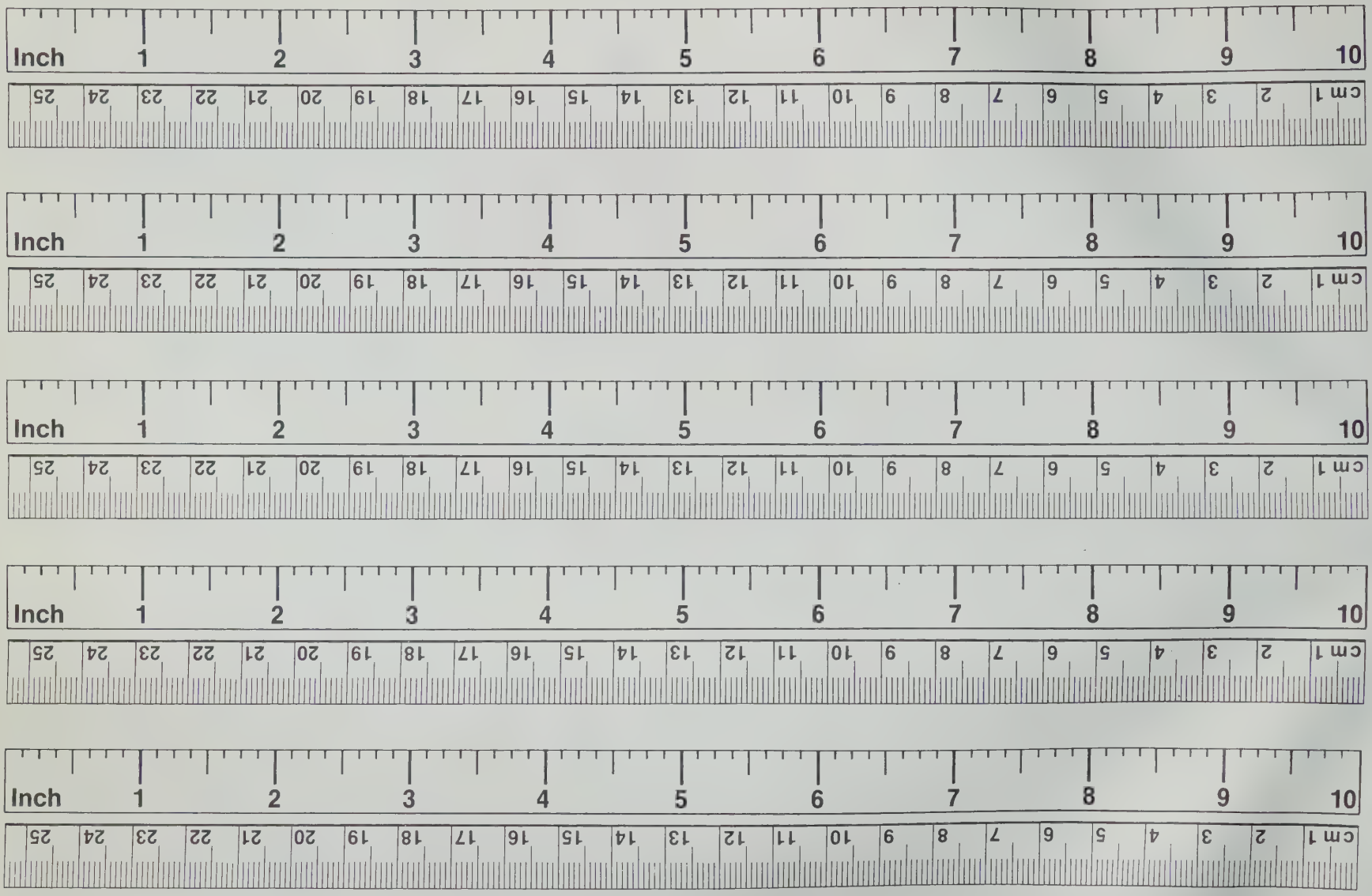
SRA products

diagnosis: an instructional aid—Mathematics Level A, SRA (1973)
Probe: L-17

Mathematics Involvement Program, SRA (1971)
Cards: 223, 283

Skill through Patterns, level 3, SRA (1974)
Spirit masters: 54, 55, 56, 66

other learning aids (described on page 216g)—Learning about Measurement, Linear Measures, Metric Trundle Wheel



10 FRACTIONS

before this chapter the learner has—

1. Associated a fraction with a region having parts marked in halves, fourths, or eighths
2. Used a number line to show addition and subtraction of whole numbers

in chapter 10 the learner is—

1. Mastering the writing of the fraction associated with a marked region or number-line model divided into same-size parts
2. Mastering the identification of the numerator and denominator of a fraction
3. Identifying a fraction with the same numerator and denominator as another name for 1
4. Comparing two fractions, using a model
5. Exploring the addition and subtraction of fractions with like denominators

in later levels the learner will—

1. Compare two fractions
2. Master the addition and subtraction of fractions with like denominators
3. Practice the renaming of fractions

Notes & Things

The emphasis in this fractions chapter is on concept development. The spirit of the chapter is one of exploration and investigation.

The concept of fractions will not be new to the learner, but no assumptions are made about any previous learning. Three different models will be presented to make sure that the understanding of fractions is not limited to pieces of pie. The pie model is effective, but the individual needs to look at many differently shaped regions, as well as at pictures of sets and at the number line too. The models used on the pupil pages will be perfectly drawn so that the youngster comes to know that a fractional part is the same size as other parts of the same model. You will have the opportunity at the start of the chapter to relate less perfect real-world models of fractions found in the child's environment.

The distinction between numeral and number is not made in this chapter. The child should come to know that, in general, whole numbers answer the question "How many?" and fractions answer the question "How much?" As a child looks at the numerator and the denominator, he will see that either of these numbers really acts as a whole number. Here is how the three models are used.

How many parts are shaded? 3
How many parts in all? 4
How much of the region is shaded? $\frac{3}{4}$

How many parts are shaded? 3
How many parts in all? 4
How much of the set is shaded? $\frac{3}{4}$

How many $\frac{1}{4}$s are covered by the arrow? 3
How many $\frac{1}{4}$s in all? 4
How much of the line is covered? $\frac{3}{4}$

The operations of addition and subtraction are introduced, but there no skill competency is expected. If any of the children recognize that we operate on fractions in very much the same way that we operate with whole numbers, you have done a fantastic job! There will be plenty of time to develop computational skills at later levels.

things

paper geometric shapes
felt or magnetic cut outs, or transparencies
labeled paper-strip models
overhead projector and circular regions divided into fractional parts
set of measuring cups
spirit master of number lines and of geometric shapes (see page 216b)

goal Think about and explore ideas through a picture clue

page 197 Can anyone guess that this is a photograph of oranges? When we want to get children thinking about fractions, there is no better place to start than with their everyday experiences.

Could one person eat an orange? Could one person share the orange with another person? How? Could 4 people share 1 orange? How? Could 8 people share 1 orange? Would each person get a very big piece? How much of the orange would 1 person get? How many things can the group think of that can be cut apart and shared.

This discussion should lead directly to the next page, where fractional parts are shown on a number line. Chances are the children won't even notice that you are changing from one model to another to show fractions.

lesson Pages 198, 199

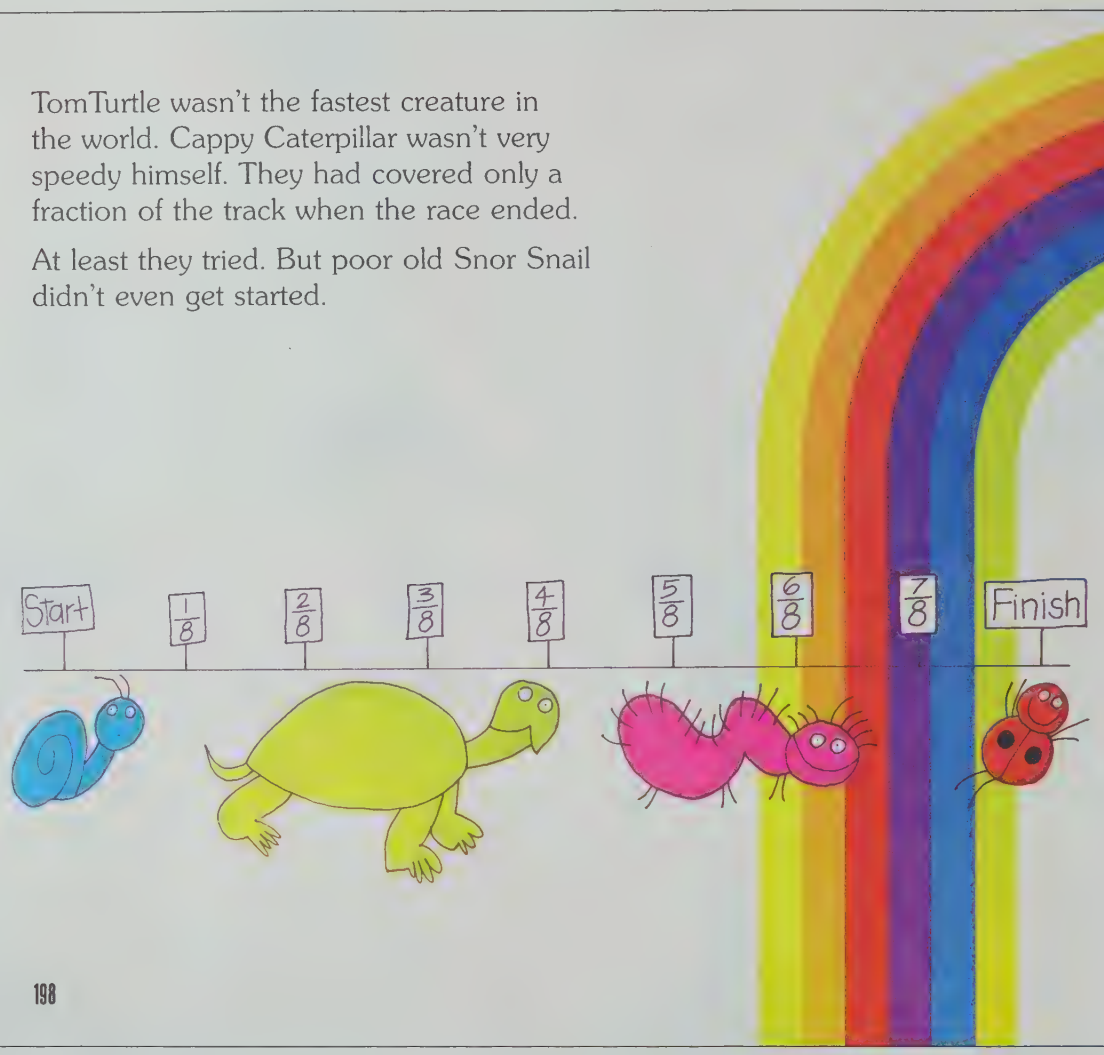
goal Relating fractions to situations

page 198 A page to talk about together. Can anyone tell how far Tom Turtle has gone? or Wiggly Worm? Don't force the answer. Settle for "They've gone part of the track" or a similar answer.

Necessity caused people to invent these strange-looking numbers called fractions. Try to discuss the part of a package of paper that remains, the amount of paint remaining in a jar, the part of a bag of candy not eaten—without using a fraction. It can be done, but many words are needed. Fractions give a more exact picture quickly.

TomTurtle wasn't the fastest creature in the world. Cappy Caterpillar wasn't very speedy himself. They had covered only a fraction of the track when the race ended.

At least they tried. But poor old Snor Snail didn't even get started.



198

Fractions tell about parts of things. Do they tell more? Your goal is to find out more about numbers called fractions.

Fractions belong in the real world, too.
Have you ever had $\frac{1}{2}$ of a glass of milk?
Maybe the milk was poured from a $\frac{1}{2}$-gallon carton.

Have you ever watched a $\frac{1}{2}$-hour TV program?
Have you ever eaten $\frac{1}{4}$ of a pizza?

goal Developing awareness of everyday uses of fractions

page 199 The everyday examples disappear after this page. The more abstract but more accurate region model and number line will be used for instructional purposes. You can always return to these pages if applications of fractions are needed for inspiration.

lesson Pages 200, 201

goal Naming a fractional part of a region

page 200 If some pupils have had no previous work with fractions, the series of questions will allow you to establish the meanings of **numerator** and **denominator.**

How many shaded? 2
How many same-size parts in all? 4
What fraction of the whole is shaded? $\frac{2}{4}$

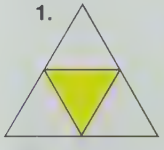
How many shaded? 3
How many same-size parts in all? 4
What fraction of the whole is shaded? $\frac{3}{4}$

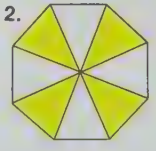
How many shaded? 3
How many same-size parts in all? 5
What fraction of the whole is shaded? $\frac{3}{5}$
How do you write the fraction? $\frac{3}{5}$

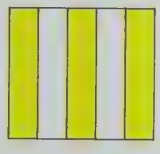
$\frac{3}{5}$ ← How many shaded parts? This number is called a numerator.
← How many parts in all? This number is called a denominator.

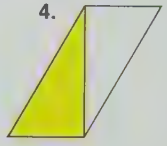
The fraction with numerator and denominator tells "HOW MUCH."

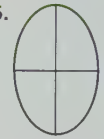
What fraction tells how much of the whole is shaded?

1.  $\frac{1}{4}$

2.  $\frac{4}{8}$ or $\frac{1}{2}$

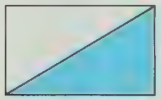
3.  $\frac{3}{5}$

4.  $\frac{1}{2}$

5.  $\frac{0}{4}$ or 0

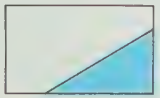
6.  $\frac{2}{2}$ or 1

200

1. 

Does this show $\frac{1}{2}$? Yes

$\frac{1}{2}$ → shaded
$\frac{1}{2}$ → in all

2. 

Does this show $\frac{1}{2}$? No

$\frac{1}{2}$ → shaded
$\frac{1}{2}$ → in all ????

It shows 1 part shaded.
BUT are the 2 parts the same size? No!

3.  Does this show $\frac{1}{2}$? Yes

4.  Does this show $\frac{1}{2}$? No

5.  Does this show $\frac{1}{4}$? Yes

6.  Does this show $\frac{1}{4}$? Yes

7.  Does this show $\frac{1}{4}$? Yes

8.  Does this show $\frac{1}{4}$? Yes

9.  Does this show $\frac{1}{4}$? No

10.  Does this show $\frac{1}{4}$? No

Talk about these.

11. Have you heard someone say they got the biggest $\frac{1}{2}$ of the candy bar? Could this be true?
If one part is bigger, it is not $\frac{1}{2}$ of the whole thing

12. If you cut a pizza to share with your friends, do you try to make the pieces the same size? Why?
Discuss. Accept answers backed by reason(s).

13. Does the denominator of a fraction tell the number of same-size parts in the whole thing? Yes
What does the numerator signal?
How many parts of the whole you are concerned with

201

goal Deciding when a part of a region shows a fraction

things paper geometric shapes

warm-up To be a fractional part, all the parts of a whole must be the same size. Cut out several geometric shapes. (Coffee filters are great for the circular regions.) Fold some into same-size parts, others into parts not of the same size. Run a flat piece of crayon over the fold lines to make them more visible. Are the parts all alike? If not, can the parts be named with a fraction? Cut apart those shapes that do not have same-size parts. Pretend that they are candy and let youngsters share. Is each one satisfied? Why not? Repeat with same-size parts.

page 201 After the **warm-up,** problems 1 through 10 can be completed independently.

lesson Pages 202, 203

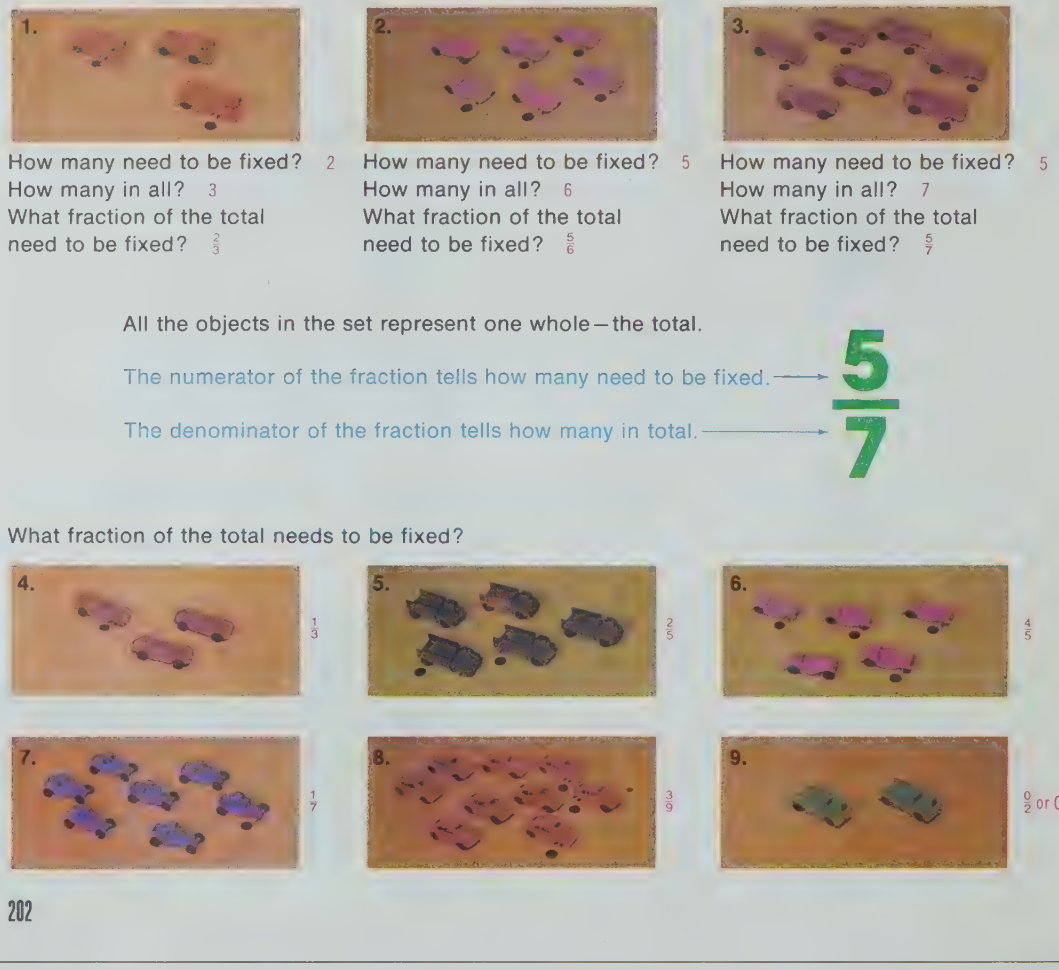
goal Naming a fractional part of a set

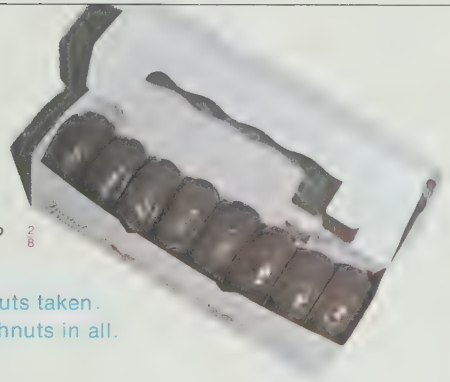
things felt or magnetic cutouts or appropriate transparencies for the overhead projector

warm-up Use whatever shapes you have available. Display a set—farm animals for example. Possible questions are:

- *How many ducks?*
- *How many animals in all?*
- *What fraction (or how much) of the whole set of animals are the ducks?*
- *What fraction (or how much) of the whole set of animals are not ducks?*

page 202 Do problems 1, 2, and 3 together, and then everyone is on his own. If anyone gets in trouble, ask the two standard questions: *How many are shaded? How many in all?*





Mike had a box of 8 doughnuts.
Jim came by and took 2.
What fraction of the doughnuts were taken? $\frac{2}{8}$

$\frac{2}{8}$ ← This number is called a numerator–doughnuts taken.
← This number is called a denominator–doughnuts in all.

Jane had a can of 3 tennis balls. 2 were brand-new.
1 had been used. Write a fraction that tells
what fraction of the balls were new. $\frac{2}{3}$

Draw a picture if it will help you answer these problems.

Sam loved animals.

1. He had a collection of 9 bugs. 4 of them were spiders. What fraction of his collection were spiders? $\frac{4}{9}$
2. He had 7 goldfish in a tank. 2 of them were black. What fraction of the goldfish were black? $\frac{2}{7}$
3. He had 4 cats, and 1 was male. What fraction of his cats were male? $\frac{1}{4}$
4. He had 6 turtles. 1 of them came from Florida. What fraction of his turtles came from Florida? $\frac{1}{6}$

What other kinds of animals do you think Sam had? Make up a story that uses a fraction to describe part of another set of Sam's pets.
Answers will vary, but do encourage creativity, originality, and humor within nonmath parts of the story.

goal Naming a fractional part of a set

page 203 The model and the example will help to review.

Consider having your pupils make illustrations for the word problems. This technique can serve as a check to find those youngsters who do or do not understand the problem itself or those who do not understand the concept. You may have to provide some help with reading.

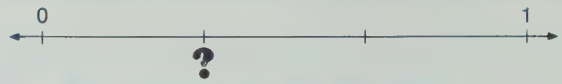
lesson Pages 204, 205, 206

goal Introduction to the number line as a model for fractional parts

page 204 Emphasize that the distances marked on a number line between 0 and 1 must all be of the same length.

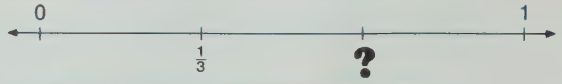
1. The distance from 0 to 1 on the number line is one unit.

2. Now the distance is divided into 3 same-size parts. The distance from 0 to ? is what fraction of the whole distance? $\frac{1}{3}$



What fraction can name this point? $\frac{1}{3}$

3. The ? has moved to a new point.



What fraction can name this point? $\frac{2}{3}$

4. The ? has moved again! That point is already named "1." Can that point have a fraction name, too? Yes

What fraction can name this point? $\frac{3}{3}$

5. The distance from 0 to $\frac{1}{3}$ is one-third. From $\frac{1}{3}$ to $\frac{2}{3}$ is also one-third. And the length from $\frac{2}{3}$ to $\frac{3}{3}$ is one-third too. How many one-thirds from 0 to 1? 3
Could $\frac{3}{3}$ be another name for 1? Yes

6. What in the world does $\frac{0}{3}$ mean on this number line? It's another name for zero.

204

1. Look at this number line.
The distance from 0 to 1 is marked in 2 same-size parts.

a A teeny, tiny bug wanted to walk from 0 to 1. He started at 0 and got to ?. He was *so* tired, he had to rest. How much of his walk had he finished? $\frac{1}{2}$
He could not decide what to do. Should he go back to 0? or should he go to 1? Which is the longer walk? Both are the same distance from $\frac{1}{2}$.
What fraction name belongs where the ? is? How many $\frac{1}{2}$s from 0 to 1? Could $\frac{2}{2}$ be another name for 1? Yes

2

2. Here's another number line.

a The distance between 0 and 1 has been marked in how many same-size parts? How much of the distance is each part? 4 $\frac{1}{2}$
Think about the fraction name for the letters marked on the number line. Remember what a fraction tells you.

b What fraction belongs at each point on the line above?
See number line above.

$\frac{1}{4}$ The numerator tells how many parts you want to talk about.

The denominator tells how many parts in all.

3. It's your turn to draw a number line.
Answers will vary according to individual choice of number line. See example.

a Make a number line that shows a distance from 0 to 1.

b You decide how many same-size parts the distance should have.

c Label the parts with fractions—the right ones, of course!

205

goal Naming points on a number line with a fraction

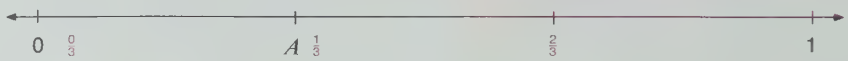
page 205 Let the questions guide the discussion. Verbal answers are all that's needed until the pupils are directed to make a number-line model. It will be a miracle if anyone does a line with comparatively accurate same-size parts. This time you will be the one to pretend a bit.

goal Practice in naming points on a number line with a fraction

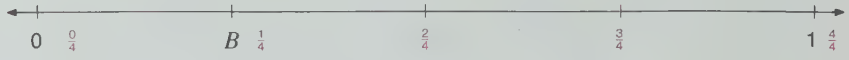
page 206 Each pupil should make his own set of number-line models. Watch for the youngster who reverses the numerator and denominator and writes the fraction upside down. Now is the time to give that individual help and encouragement.

Write a fraction for each point labeled with a letter. Note: For this problem, student labels points named by letters. Remaining answers are for problem 6.

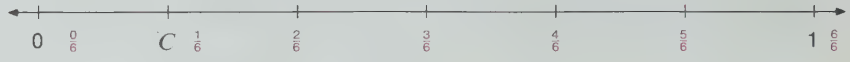
1.



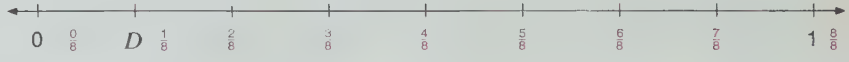
2.



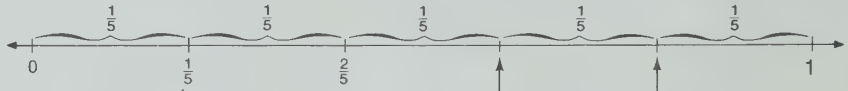
3.



4.



5. Make sure you know how we can give names to all points on a number line.
This distance is divided into 5 same-size parts.



This is $\frac{1}{5}$ of the distance

This is $\frac{2}{5}$ of the distance

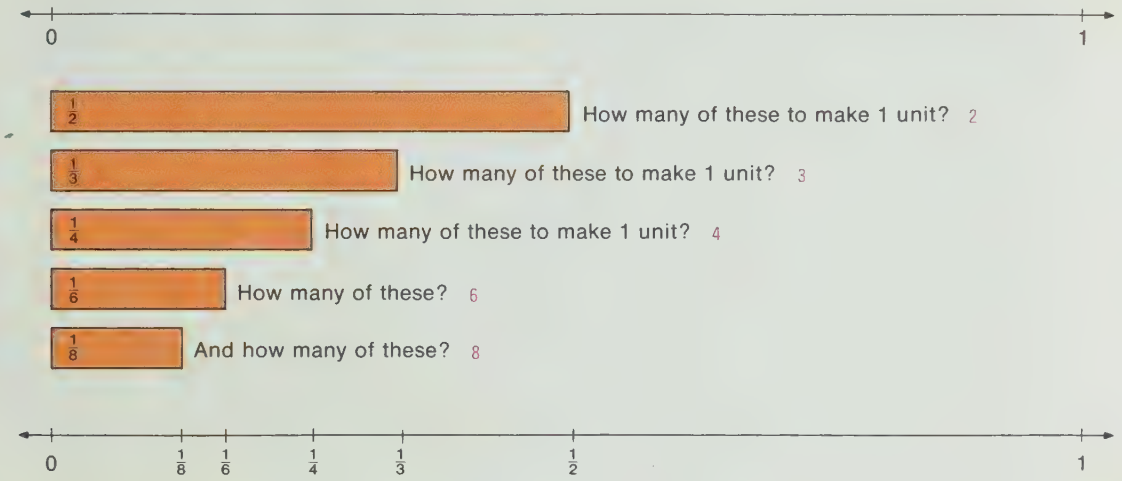
a How many $\frac{1}{5}$s from 0 to this point? 3

b What fraction names the next point? $\frac{4}{5}$

c How many $\frac{1}{5}$s from 0 to 1? 5 Is $\frac{5}{5}$ another name for 1? Yes

6. Look back. Copy each number line on your paper.
Write the fraction that names each point. See number lines for problems 1 through 4.

206



Is the first fraction greater than, less than, or equal to the second fraction?
Use the number line for help. Should $>$, $<$, or $=$ be in each ring?

1. $\frac{1}{3} \overset{>}{?} \frac{1}{6}$	**2.** $\frac{1}{8} \overset{<}{?} \frac{1}{2}$	**3.** $\frac{1}{4} \overset{>}{?} \frac{1}{6}$	**4.** $\frac{1}{2} \overset{>}{?} \frac{1}{3}$	**5.** $\frac{1}{6} \overset{<}{?} \frac{1}{3}$
6. $\frac{1}{2} \overset{>}{?} \frac{1}{4}$	**7.** $1 \overset{>}{?} \frac{1}{8}$	**8.** $\frac{1}{2} \overset{>}{?} \frac{1}{6}$	**9.** $\frac{1}{6} \overset{>}{?} 0$	**10.** $\frac{1}{3} \overset{<}{?} \frac{1}{2}$
11. $\frac{1}{8} \overset{=}{?} \frac{1}{8}$	**12.** $\frac{1}{4} \overset{>}{?} \frac{1}{8}$	**13.** $\frac{1}{4} \overset{<}{?} \frac{1}{3}$	**14.** $\frac{1}{6} \overset{>}{?} \frac{1}{8}$	**15.** $\frac{1}{8} \overset{<}{?} \frac{1}{3}$

Sam bought a sandwich. It was cut into 4 same-size parts.
He ate 2 parts. Did he eat less than $\frac{1}{2}$, more than $\frac{1}{2}$,
or just $\frac{1}{2}$ of the sandwich?

207

lesson Pages 207, 208, 209, 210

goal Comparing two fractions

things labeled paper-strip models

page 207 The youngsters may use either the number-line model or the paper strips (rectangular models) to help them make comparisons. Note that all fractions have a numerator of 1.

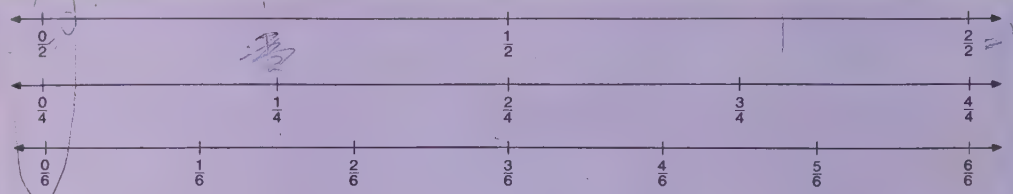
Check youngsters who have difficulty. Can they read the relation signs correctly? If not, better review the signs by writing them on the board to help avoid confusion.

$>$	$<$
greater	less

Provide sets of labeled paper-strip models that can be manipulated in making comparisons.

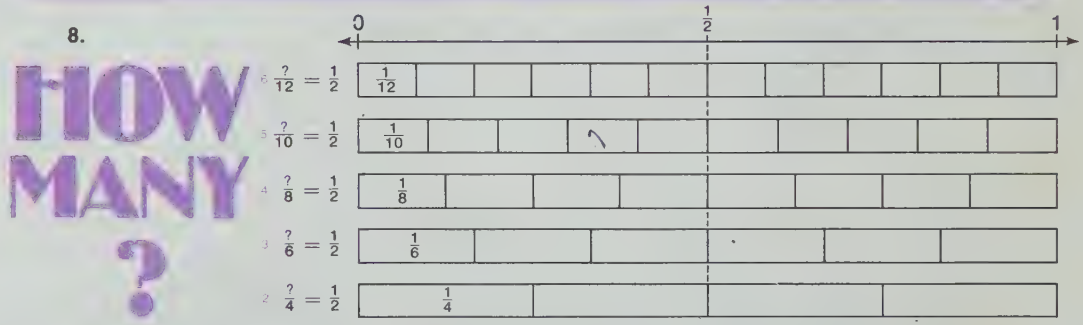
goal Finding other fraction names for the fraction $\frac{1}{2}$

page 208 If you read the sequence of questions on the pupil page, the children can be free to focus on the number lines. The assumption that the children know $\frac{0}{2}$ and $\frac{0}{4}$ are each another name for 0 is made on this page. Please make sure this assumption is true, or everyone will have trouble with the questions.



1. Are the distances from 0 to $\frac{1}{2}$ and from $\frac{1}{2}$ to 1 the same?
2. Is $\frac{2}{2}$ another name for 1?
3. Are the distances from 0 to $\frac{1}{4}$ and from 0 to $\frac{1}{2}$ the same?
4. Are the distances from 0 to $\frac{2}{4}$ and from $\frac{2}{4}$ to $\frac{4}{4}$ the same?
5. Are the distances from 0 to $\frac{1}{2}$ and from 0 to $\frac{2}{4}$ the same? Does $\frac{1}{2} = \frac{2}{4}$?
6. Does $\frac{1}{2} = \frac{3}{6}$? Look at the two distances on the number lines above.
7. Are the distances from 0 to $\frac{2}{4}$ and from 0 to $\frac{3}{6}$ the same? Does $\frac{2}{4} = \frac{3}{6}$?

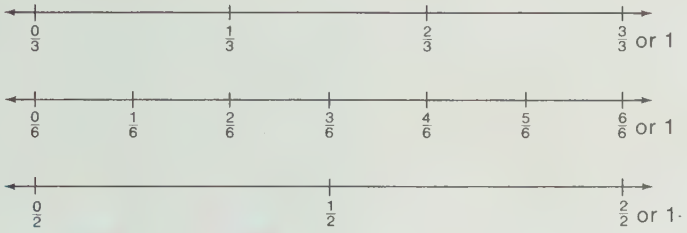
8.



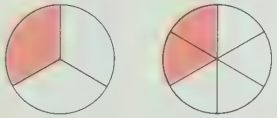
You have found many fraction names for 1.
Now you have many fraction names for $\frac{1}{2}$ too!

208

Let's go on with the name hunt.

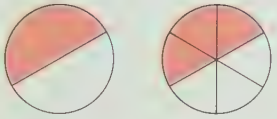


1. Does $\frac{1}{3} = \frac{2}{6}$? Yes
Check by looking at this model.



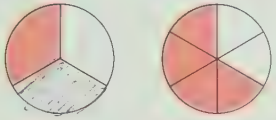
Are you sure?

2. Does $\frac{1}{2} = \frac{3}{6}$? Yes
Check by looking at this model.



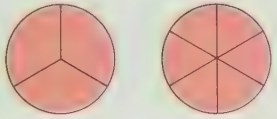
Are you sure?

3. Does $\frac{2}{3} = \frac{4}{6}$? Yes
The number line and the models say yes.



Do you agree?

4. Does $\frac{3}{3} = \frac{6}{6}$? Yes



5. Draw models to show that $\frac{2}{2} = \frac{3}{3}$.

209

goal Renaming fractions

page 209 The pupils have to pretend that they can move one line on top of another. *If the line with thirds and the line with sixths were slid together, would the point marked $\frac{0}{3}$ and the point marked $\frac{0}{6}$ match? Would the point marked $\frac{1}{3}$ match the point marked $\frac{2}{6}$?* Continue with this line of questioning.

It may be necessary to use flannel pieces to **prove** $\frac{1}{3} = \frac{2}{6}$. There is another way: Have each pupil trace one model and put it over the top of the second model to make sure that the two shaded parts match.

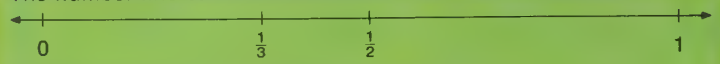
The pupils can use a number line, a circular region, or a rectangular region for problem 5. Look out! There will be trouble if a child tries to fold a region into thirds. This is a hard job with a rectangular region or number line, and it is nearly impossible with a circular region.

goal Comparing two fractions, using a number-line model

page 210 Encourage the use of the number line in making comparisons. Mastery of this skill is not expected at this time.

The problems should be independent work. Require pupils to write only the symbol.

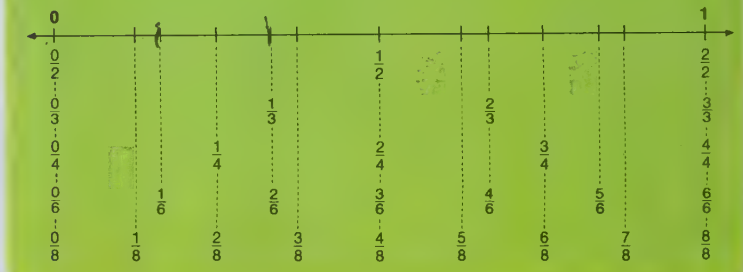
The number line can be used to show the order of fractions.



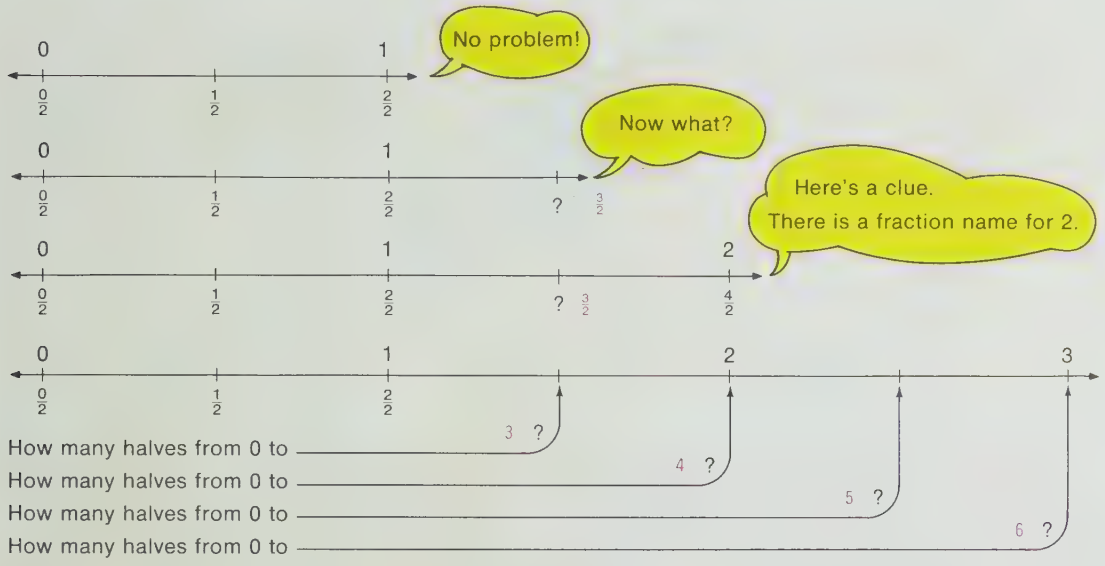
Which is greater, $\frac{1}{2}$ or $\frac{1}{3}$? Which is less, $\frac{1}{2}$ or $\frac{1}{3}$?

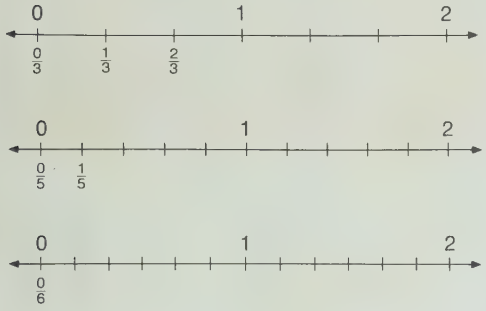
Should $>$, $<$, or $=$ be in each ring?
The number line below might help.

1. $\frac{1}{3} \overset{>}{?} \frac{1}{6}$	2. $\frac{5}{8} \overset{>}{?} \frac{1}{2}$	3. $\frac{3}{4} \overset{<}{?} \frac{5}{6}$
4. $\frac{1}{2} \overset{<}{?} \frac{2}{3}$	5. $\frac{4}{6} \overset{=}{?} \frac{2}{3}$	6. $\frac{1}{2} \overset{=}{?} \frac{2}{4}$
7. $1 \overset{>}{?} \frac{7}{8}$	8. $\frac{1}{2} \overset{>}{?} \frac{1}{6}$	9. $\frac{1}{6} \overset{=}{?} \frac{1}{6}$
10. $\frac{1}{6} \overset{>}{?} 0$	11. $\frac{2}{3} \overset{>}{?} \frac{1}{2}$	12. $\frac{7}{8} \overset{>}{?} \frac{5}{8}$
13. $\frac{1}{4} \overset{=}{?} \frac{2}{8}$	14. $\frac{2}{3} \overset{=}{?} \frac{4}{6}$	15. $\frac{5}{6} \overset{>}{?} \frac{6}{8}$
16. $0 \overset{<}{?} \frac{1}{3}$	17. $\frac{3}{4} \overset{<}{?} 1$	18. $0 \overset{<}{?} 1$



210





How many thirds from 0 to 1? 3
That's another name for 1!
How many thirds from 0 to 2? 6
That's another name for 2!

How many fifths from 0 to 1? 5
That's another name for 1!
How many fifths from 0 to 2? 10
That's another name for 2!

What's another name for 1 on this line? $\frac{6}{6}$
And another name for 2? $\frac{12}{6}$
What would be another name for 3? $\frac{18}{6}$

211

lesson Page 211

goal Introduction of fraction names for numbers greater than 1

memo This page is purely for exploration. Pupils should not think that fractions name only parts less than or equal to 1.

page 211 The number line is extended from 1 unit to 2, and then to 3 units. The units are each divided into fractional parts. The segments formed are counted and the points labeled. The total number of parts to a given point are counted. This number is written as a numerator. The number of parts in **1 unit** is the denominator.

Don't worry if a couple of children fight this idea. They will have lots more experience in the next level. That's soon enough for those who aren't ready now.

lesson Pages 212, 213, 214, 215

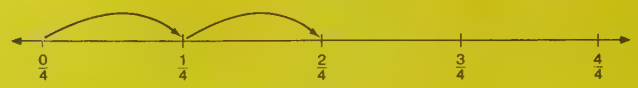
goal Introduction of the addition of fractions with like denominators

memo The addition of fractions will be introduced with a number-line model first and then with a region model.

things spirit master of number lines

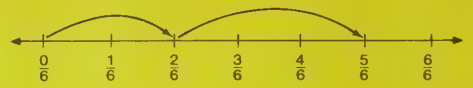
page 212 Work through the number-line examples with all the children. A spirit master of number lines may be needed to complete the last problems. It will also be useful for extra practice.

1. You can add whole numbers. $3 + 2 = 5$
Can you add fractions? $\frac{1}{4} + \frac{1}{4} = \underline{?}$ A number line will help.



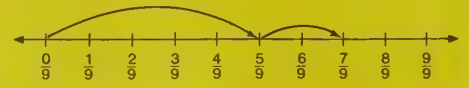
$\frac{1}{4} + \frac{1}{4} = \frac{2}{4}$

a



$\frac{2}{6} + \frac{3}{6} = \underline{?}$ $\frac{5}{6}$

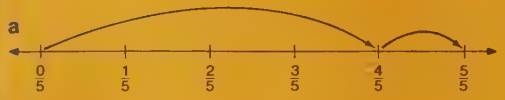
b



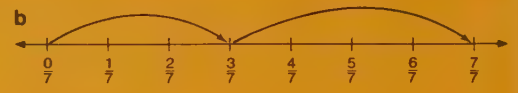
$\frac{5}{9} + \frac{2}{9} = \underline{?}$ $\frac{7}{9}$

2. Write the addition problem shown on each number line.

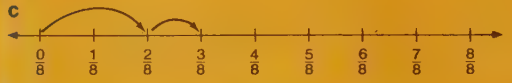
a



b



c



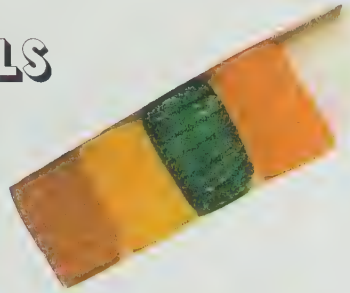
d

Try some more

3. $\frac{1}{6} + \frac{4}{6}$ $\frac{5}{6}$
4. $\frac{1}{9} + \frac{4}{9}$ $\frac{5}{9}$
5. $\frac{3}{5} + \frac{1}{5}$ $\frac{4}{5}$
6. $\frac{3}{8} + \frac{4}{8}$ $\frac{7}{8}$
7. $\frac{2}{3} + \frac{1}{3}$ $\frac{3}{3}$ or 1
8. $\frac{1}{2} + \frac{1}{2}$ $\frac{2}{2}$ or 1

212

We can use other **MODELS** to show addition too. This region has been separated into 5 parts.



$\frac{3}{5} + \frac{1}{5} = ?$

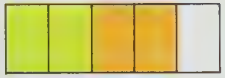
Now try these

1. 

$\frac{1}{4} + \frac{2}{4} = \underline{?}$ $\frac{3}{4}$

2. 

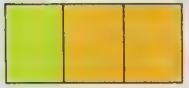
$\frac{3}{6} + \frac{2}{6} = \underline{?}$ $\frac{5}{6}$

3. 

$\frac{2}{5} + \frac{2}{5} = \underline{?}$ $\frac{4}{5}$

4. 

$\frac{2}{8} + \frac{3}{8} = \underline{?}$ $\frac{5}{8}$

5. 

$\frac{1}{3} + \frac{2}{3} = \underline{?}$ $\frac{3}{3}$ or 1

6. 

$\frac{1}{2} + \frac{1}{2} = ?$ $\frac{2}{2}$ or 1

Draw a number line or a region if you need help on these.

7. $\frac{1}{5} + \frac{2}{5} = \underline{?}$ $\frac{3}{5}$
8. $\frac{1}{5} + \frac{1}{5} = \underline{?}$ $\frac{2}{5}$
9. $\frac{3}{8} + \frac{1}{8} = \underline{?}$ $\frac{4}{8}$
10. $\frac{1}{6} + \frac{1}{6} = \underline{?}$ $\frac{2}{6}$
11. $\frac{3}{9} + \frac{4}{9} = \underline{?}$ $\frac{7}{9}$
12. $\frac{5}{7} + \frac{1}{7} = \underline{?}$ $\frac{6}{7}$
13. $\frac{5}{8} + \frac{2}{8} = \underline{?}$ $\frac{7}{8}$
14. $\frac{0}{4} + \frac{1}{4} = \underline{?}$ $\frac{1}{4}$
15. $\frac{3}{4} + \frac{3}{4} = \underline{?}$ $\frac{6}{4}$
16. $\frac{3}{5} + \frac{2}{5} = \underline{?}$ $\frac{5}{5}$ or 1

213

goal Exploring the addition of fractions, using a region model

warm-up You may want to flip back to page 200 and use those models to get ready for addition. Each model also names two fractions: the shaded part, the plain part. Together they make the whole figure. *How much is shaded? How much is not shaded? How much in all?* This line of questioning will help you develop readiness for the addition of fractions.

page 213 Study problems 1 through 6 together. Verbal answers are sufficient.

If the pupils seem confident, ask them to try to complete problems 7 through 16 independently. Encourage them to use models when in doubt.

goal Introduction to the subtraction of fractions with like denominators

things circular regions divided into fractional parts
spirit master of number lines

page 214 Here the subtraction of fractions is introduced as a taste of what is to come. Do the examples together. Encourage learners to make number lines if necessary.

Circular regions are a bit tricky to present on paper, but they can be very effective in an activity. Show all the pieces together as 1 whole circular region to start.

Here are $\frac{6}{6}$.
Let's take $\frac{1}{6}$ *away.*
Remove a $\frac{1}{6}$ piece and put it aside.
What fractional part remains?

O.K. We have $\frac{5}{6}$. *Can we take away* $\frac{2}{6}$?
Remove two $\frac{1}{6}$ pieces.
What fractional part remains?

You should always start with the whole circular region; otherwise the pupils will not have a basis for determining what fractional part one piece represents.

If I had $\frac{3}{4}$, could I give you $\frac{1}{4}$? Yes
How much would I have left? $\frac{2}{4}$

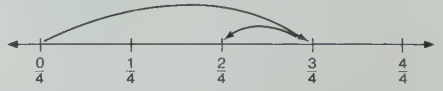
If you had $\frac{2}{3}$, could I have $\frac{1}{3}$? Yes
How much would you have left? $\frac{1}{3}$

If they had $\frac{4}{5}$, could we have $\frac{2}{5}$? Yes
How much would they have left? $\frac{2}{5}$

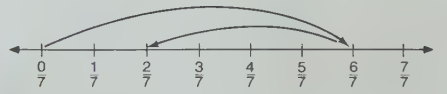
If you could answer those three questions, you know how to

SUBTRACT

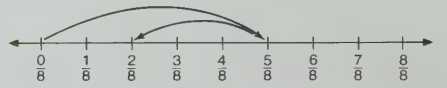
If you couldn't answer those questions, don't feel blue. This page will help you.



This shows $\frac{3}{4} - \frac{1}{4}$. What remains? $\frac{2}{4}$



This shows $\frac{6}{7} - \frac{4}{7}$. What remains? $\frac{2}{7}$



This shows $\frac{5}{8} - \frac{3}{8}$. What remains? $\frac{2}{8}$

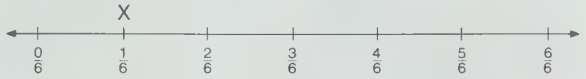
Try these.

1. $\frac{5}{6} - \frac{1}{6}$ $\frac{4}{6}$ **2.** $\frac{2}{3} - \frac{1}{3}$ $\frac{1}{3}$ **3.** $\frac{4}{9} - \frac{2}{9}$ $\frac{2}{9}$ **4.** $\frac{7}{9} - \frac{5}{9}$ $\frac{2}{9}$

5. $\frac{4}{4} - \frac{1}{4}$ $\frac{3}{4}$ **6.** $\frac{4}{5} - \frac{3}{5}$ $\frac{1}{5}$ **7.** $\frac{7}{8} - \frac{6}{8}$ $\frac{1}{8}$ **8.** $\frac{6}{7} - \frac{4}{7}$ $\frac{2}{7}$

214

1. There was to be a mile-long relay race.
There were to be 6 runners.
Each runner was to run $\frac{1}{6}$ of a mile.



The first runner finished his part of the race.
How much more of the mile had to be covered? $\frac{5}{6}$
How many more runners had to run? 5

Draw a picture if you need help on these.

2. Bill took $\frac{1}{8}$ of the cake.
His big brother took $\frac{3}{8}$ of the cake.
How much was taken by Bill and his brother? $\frac{4}{8}$
How much of the cake was left? $\frac{4}{8}$

3. June put $\frac{1}{4}$ of the stuff away.
Her mom put another $\frac{1}{4}$ of it away.
How much was put away? $\frac{2}{4}$
How much was left? $\frac{2}{4}$

4. The horse got $\frac{1}{2}$ of the bale of hay.
The cow got $\frac{1}{2}$ of the same bale of hay.
How much of the bale of hay was used? $\frac{2}{2}$ (or all of it)
How much of the bale was left? $\frac{0}{2}$ (or none of it)

5. He used $\frac{3}{8}$ of the ribbon for his package.
And she used $\frac{5}{8}$ of the ribbon for her package.
How much of the ribbon was used? $\frac{8}{8}$ (or all of it)
How much of the ribbon was left for me to use? $\frac{0}{8}$ (or none of it)

6. The first night he read $\frac{1}{3}$ of the book.
The next night he read another $\frac{1}{3}$.
How much had he read by then? $\frac{2}{3}$
How much more did he have to read? $\frac{1}{3}$

215

goal Using fractions in word problems

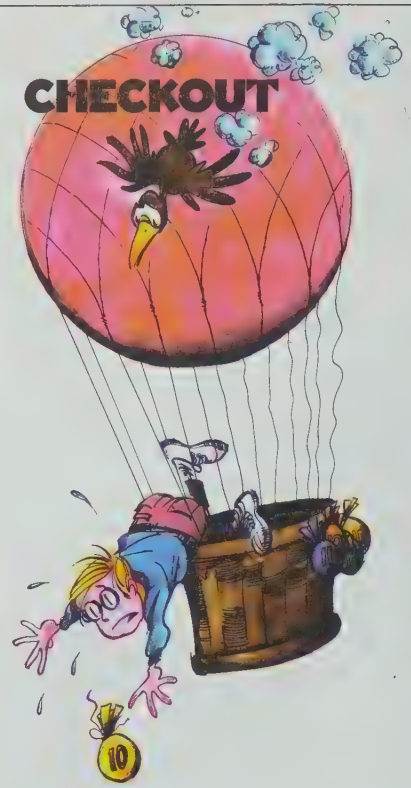
page 215 Share ideas in these problem situations. Each problem leads to operating intuitively. Verbal answers are all that you want. There certainly is no need to write math sentences. However, encourage the youngsters to make illustrations—not only to check on comprehension but to verify their answers.

lesson Page 216

goal **Checkout**—naming a fractional part of a region; comparing two fractions

memo Mastery of the addition and subtraction of fractions is not expected at this level.

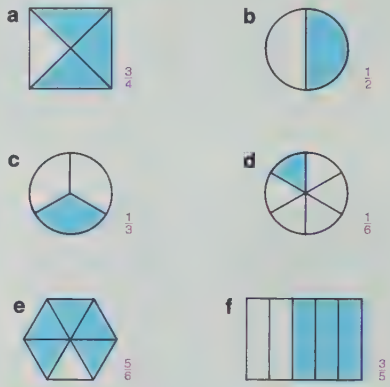
page 216 Having a pupil write the fraction associated with the shaded parts of each region in problem 1 will let you know how confident he is with the meanings of **numerator** and **denominator.** After this task is done, ask him to go back and pretend that the whole region is shaded. *Now write the fraction that names each whole region.*



216

Skill: Writing fractions shown by the shaded part of regions

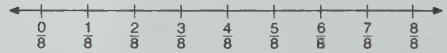
1. What fraction of the whole figure is shaded?



2. Should >, <, or = be in each ring? Skill: Comparing fractions

a $\frac{1}{2}$ (?) $\frac{1}{4}$ b $\frac{6}{8}$ (?) $\frac{7}{8}$ c $\frac{2}{2}$ (?) 1 d $\frac{8}{8}$ (?) $\frac{5}{5}$

*3. Add. Use the number line if you need it.



a $\frac{2}{8} + \frac{3}{8}$ $\frac{5}{8}$ b $\frac{1}{8} + \frac{5}{8}$ $\frac{6}{8}$ c $\frac{3}{8} + \frac{2}{8}$ $\frac{5}{8}$ d $\frac{4}{8} + \frac{4}{8}$ $\frac{8}{8}$ or 1

Skill: Subtraction of fractions with common denominators

*4. Subtract. Use the number line if you need it.

a $\frac{2}{5} - \frac{1}{5}$ $\frac{1}{5}$ b $\frac{4}{5} - \frac{2}{5}$ $\frac{2}{5}$ c $\frac{5}{5} - \frac{3}{5}$ $\frac{2}{5}$ d $\frac{3}{5} - \frac{2}{5}$ $\frac{1}{5}$

See activity 1, page 216a.

See activity 2, page 216a.

RESOURCES

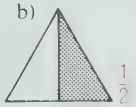
another form of evaluation

for Checkout – page 216

1. What fraction of the whole figure is shaded?

a) $\frac{1}{3}$

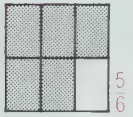
b) $\frac{1}{2}$



c) $\frac{1}{6}$

d) $\frac{3}{4}$

e) $\frac{5}{6}$



f) $\frac{3}{5}$

*2. Add. Use the number line if you need it.

a) $\frac{3}{8} + \frac{3}{8} = \frac{6}{8}$

b) $\frac{5}{8} + \frac{3}{8} = \frac{8}{8}$ or 1

c) $\frac{2}{8} + \frac{3}{8} = \frac{5}{8}$

d) $\frac{7}{8} + \frac{1}{8} = \frac{8}{8}$ or 1

*3. Subtract. Use the number line if you need it.

a) $\frac{3}{4} - \frac{1}{4} = \frac{2}{4}$

b) $\frac{4}{4} - \frac{3}{4} = \frac{1}{4}$

c) $\frac{2}{4} - \frac{1}{4} = \frac{1}{4}$

d) $\frac{3}{4} - \frac{3}{4} = \frac{0}{4}$

activities

1. things set of measuring cups, water

Most youngsters love to play with water. A set of measuring cups and a pan of water make a dramatically different model—good for any child who simply isn't "with" the notion of fractions. The cups will provide experiences with both the fraction concept and the operations of addition and subtraction.

2. things paper squares of different colors, paste, scissors, sheets of background paper

Art class is a perfect place to enjoy paper mosaics. The pieces for the mosaic will, of course, be fractional parts of a region. Cut each square into four same-size parts.
Or use six-sided regions. Use any other shapes that can be divided into same-size parts.

The pieces can be arranged on a sheet of paper and then pasted down. Encourage color mixing.

additional learning aids

concept—chapter objectives 1, 3, 4, 5

SRA products

Computapes, SRA (1972)
Module 5, Lessons: FR 1, 2, 3, 4
diagnosis: an instructional aid—Mathematics Level A, SRA (1973)
Probe: L-9
Mathematics Involvement Program, SRA (1971)
Cards: 253, 224, 234
Visual Approach to Mathematics, level 3, SRA (1967)
Visuals: 23, 26

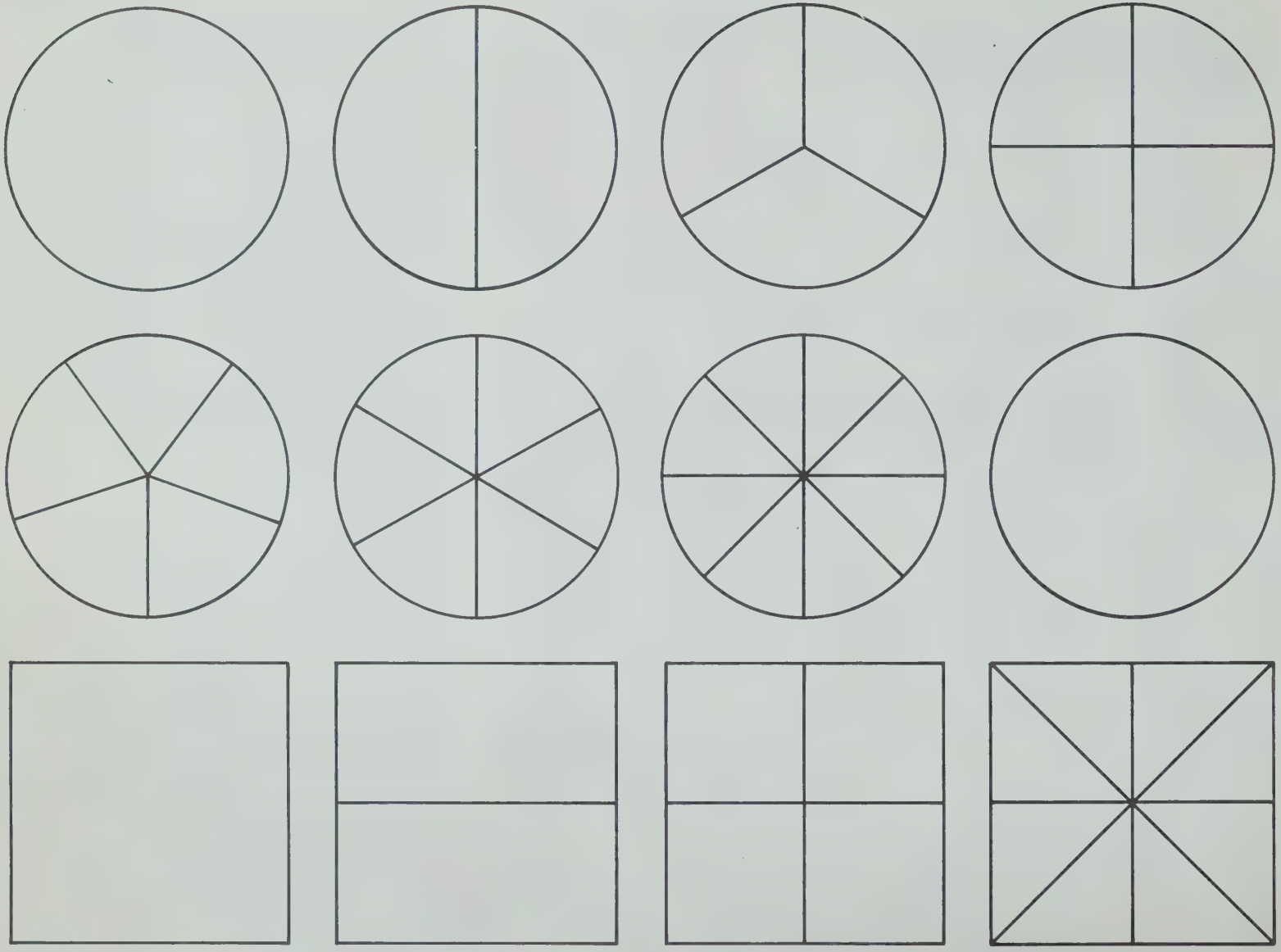
other learning aids (described on page 216g)—Cuisenaire rods, Fraction Bars Student Activity Book, Fraction Dominoes, Fraction Inlay Boards, Fraction Line Set

notation—chapter objective 2

SRA products

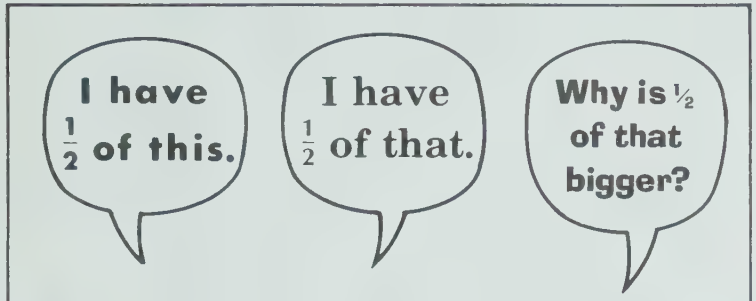
Mathematics Involvement Program, SRA (1971)
Cards: 323, 185

name_____

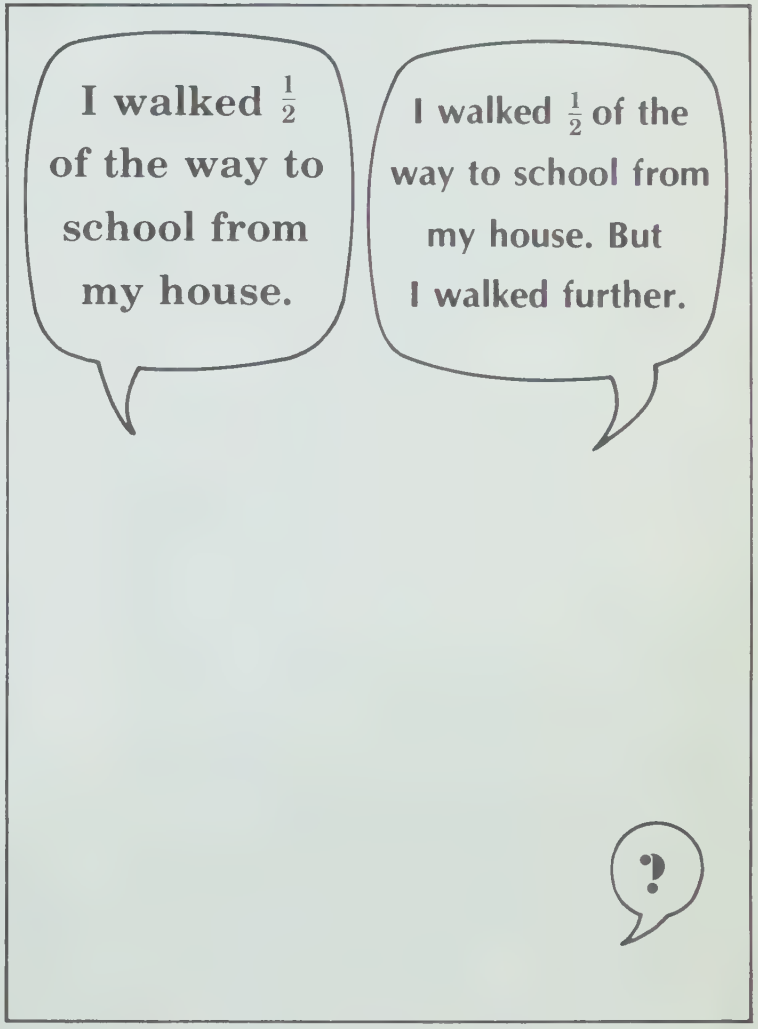


name _____

What will it look like?

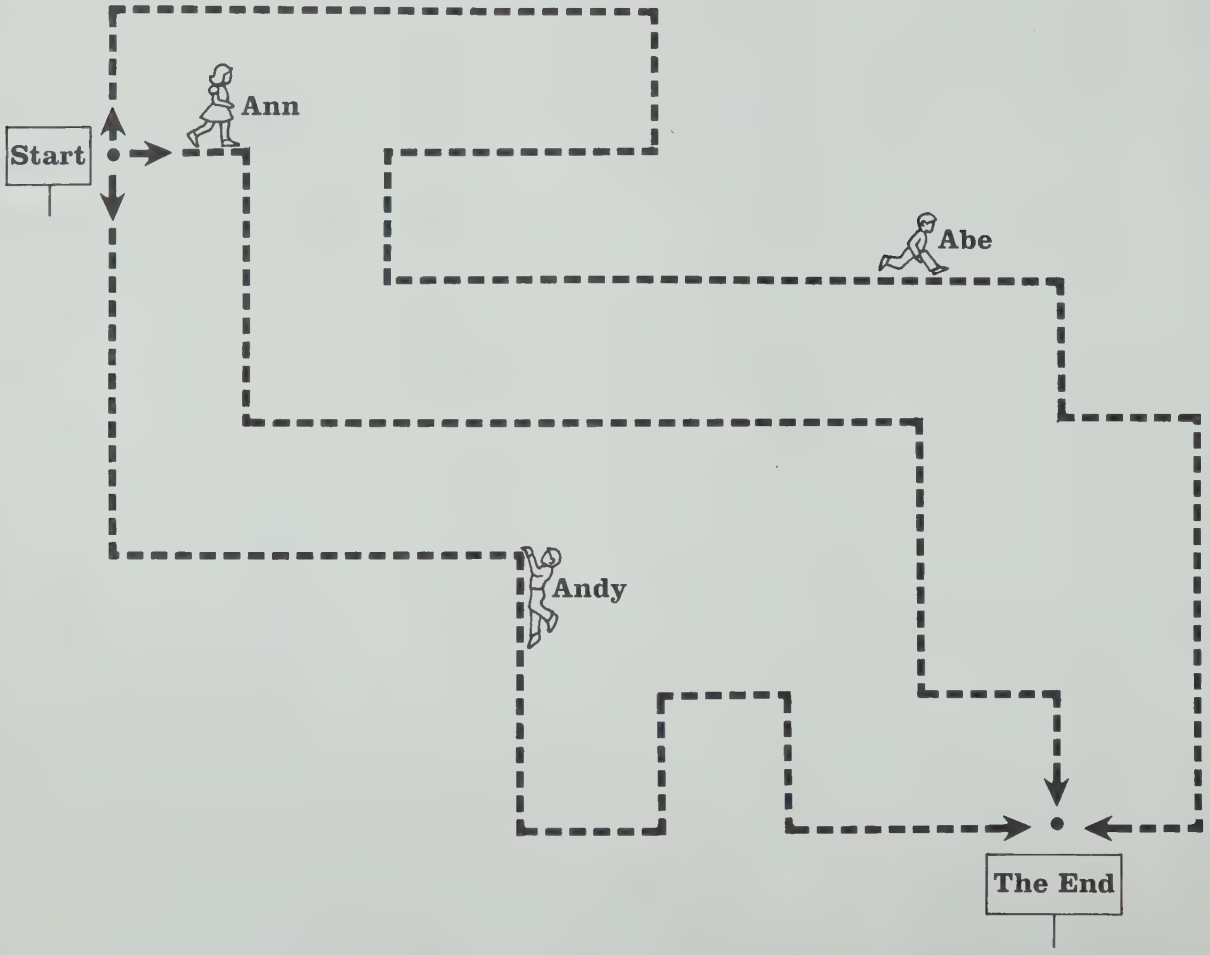


Draw them with their map.



name _____

Use a cm ruler.
Measure. Whose path is longest?



0
5

CE!

school use.

Other Learning Aids

whole-number concepts

Abacus board (Creative Publications) Counting board useful for teaching place value
Abacus Spinner Game (Math Shop) Game for recognition and understanding of place value
Chip Trading (Scott Scientific) Several games that develop understanding of place value
Flip-A-Strip Place Value Chart (Developmental Learning Materials) Easel board chart to provide practice with place value and whole-number operations
Place Value I and II (Math Shop) Self-correcting cards to provide practice in reading of numbers through hundred millions

whole-number operations

Calculator (Sigma Scientific) Four place-value calculator for addition and subtraction
Checkermatics (SEE) [level 1] A game for reinforcement and enrichment in whole-number skills
Counting chips (Creative Publications) Plastic disks of eight colors
Cuisenaire Rods® (Cuisenaire) Centimetre rods that provide a concrete approach to developing operations
Dividing Machine (DLM) Device to reinforce basic division facts
Fact-O (Math Shop) Activities for addition and subtraction facts
I Win (Scott, Foresman) [sets 1 and 2] Cards to provide practice in four basic operations
Lots-A-Links (Creative Publications) Plastic links that snap together for counting activities
Mathematical Balance (Mind/Matter) Plastic balance and weights to show relationships between addition and subtraction
Mathfacts Games™ (Milton Bradley) [levels 3–5] Self-instructional and self-checking games that deal with the basic facts
Multifax & Quotient (Math Shop) Games in which number sentences are formed
Multiplying Machine (Math Shop) Self-checking machine used to practice the facts
Orbiting the Earth (Scott, Foresman) [addition and subtraction] Game with vinyl playing field for fact practice
Quizmo (Milton Bradley) Lotto-type game that reinforces basic skills with whole numbers
Stamina (Creative Publications) Spinner game providing practice with whole-number operations
Tally and Difference (Creative Publications) Games in which the addition and subtraction facts are used to form number sentences
Unifix Mathematics Classroom Kit (ETA) Set of interlocking cubes that develop understanding of the whole-number operations
Veri-Tech Senior (ETA) [addition, subtraction, and multiplication books] A self-checking device that provides practice with whole-number operations
Winning Touch (Ideal) A game for multiplication facts

fractional-number concepts

Action Fraction Games (Constructive Playthings) Games to develop concepts and skill with fractions
Fraction Bars Student Activity Book (Creative Publications) Games and activities to teach specific objectives for fractions
Fraction Dominoes (SEE) Game involving matching a fractional numeral with its model
Fraction Inlay Boards (General Learning Corp.) Fractional parts of circles or squares
Fraction Line Set (Sigma Scientific) An activity to help visualize operations by computing with fraction strips

geometry

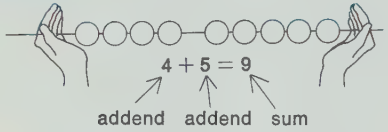
Geoboard Activity Cards (Creative Publications) [primary and intermediate sets] Geoboard activities that lead pupils to explore, compare, and make shapes
Geoboard Kit (Cuisenaire) Plastic geoboards and related activity cards that show basic geometric concepts
Learn to Fold—Fold to Learn (Lyons & Carnahan) Workbook that presents a variety of activities to demonstrate symmetry
Metric Primary Shapes (Invicta) Basic geometric shapes and work cards that reinforce identification skills
Mira (Creative Publications) An aid for investigating properties of plane geometry
Mira Math for Elementary School (Creative Publications) Series of activities to be used with the mira
Paper and Pencil Geometry (Lyons & Carnahan) A geometry book that provides activities to develop basic geometric concepts
Pattern Blocks and Mirrors (Math Shop) Colored blocks used to investigate geometric figures and mirror reflections
Tangrams (Creative Publications) A 7-piece puzzle to be used with tangram shapes
Tangramath (Creative Publications) Book to be used with tangram pieces for exploring concepts of shape, congruence, similarity, and area

measurement

Equal Pan Scales (ESA) Pan balance designed to develop weighing techniques
Judy Clock (General Learning) Movable hands and visible functioning gears to provide learning experiences in telling time
Learning about Measurement (Lyons & Carnahan) A workbook for customary and metric measurement activities
Linear Measures (DLM) A series of linear measuring activities using the customary system
Making and Using Graphs and Nomographs (Lyons & Carnahan) A workbook that develops concepts and skill in the making and reading of graphs
Metric Trundle Wheel (Invicta) Wheel designed to introduce concepts of linear measurement
100 g Balance (ESA) A plastic balance for weighing masses of between 1 gram and 100 grams
Tell-Time Quizmo (Milton Bradley) A lotto-type game to stimulate interests in telling time

GLOSSARY

addition Putting together

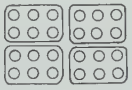


array

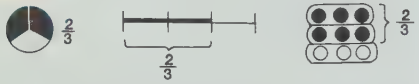
digit Any of the symbols 0, 1, 2, 3, 4, 5, 6, 7, 8, 9

division

$24 \div 6 = 4$ or $6\overline{)24}$ (4, 24, 24 (4 × 6), 0)



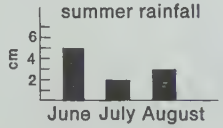
fraction A number that tells how much



$\frac{2}{3}$ ← numerator (how many parts shaded)
$\frac{2}{3}$ ← denominator (how many parts in all)

graph A way of picturing information

a bar graph:

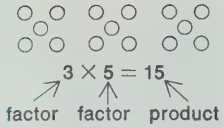


line segment A part of a line

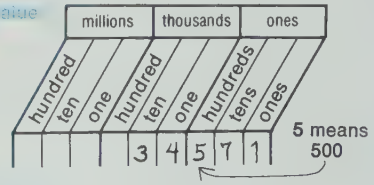
math sentence

addition: $3 + 4 = 7$
subtraction: $16 - 7 = 9$
missing factor: $3 \times \square = 12$
division: $12 \div 3 = 4$

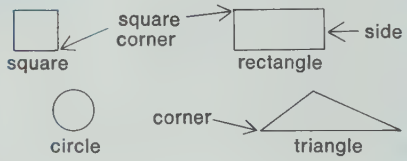
multiplication



place value

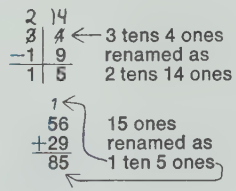


plane figure

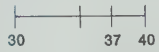


region

renaming

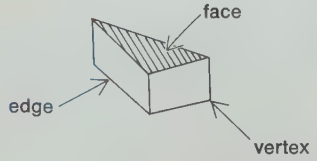


rounding



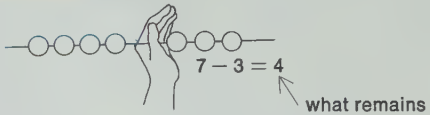
37 is closer to 40 than to 30. 37 rounds to 40.

solid object



standard unit of measure An agreed-upon length, weight, or capacity used to measure things. (See Tables of Measure.)

subtraction Taking away



symbol

+	plus	>	is greater than
−	minus	<	is less than
×	times	=	equals
÷	divided by	) (	division problem

symmetry Fold along the line. The two parts match.

tally chart A way to record a count

Games Won

Joe	𝍸 |
Sam	𝍸 ||

tally marks

whole number Tells how many
0, 1, 2, 3, 4, 5, and the rest of the counting numbers
even numbers—0, 2, 4, 6, and so on
odd numbers—1, 3, 5, 7, and so on

INDEX

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TABLES OF MEASURE

LENGTH

CUSTOMARY		METRIC	
12 inches (in.)	= 1 foot	10 millimetres (mm)	= 1 centimetre
3 feet (ft.)	= 1 yard	10 centimetres (cm)	= 1 decimetre
1760 yards (yd.)	= 1 mile	10 decimetres (dm)	= 1 metre
		10 metres (m)	= 1 dekametre
		10 dekametres (dkm)	= 1 hectometre
		10 hectometres (hm)	= 1 kilometre (km)

AREA

144 square inches (sq. in.)	= 1 square foot	100 square millimetres (sq. mm)	= 1 square centimetre
9 square feet (sq. ft.)	= 1 square yard	10,000 square centimetres (sq. cm)	= 1 square metre
4840 square yards (sq. yd.)	= 1 acre	100 square metres (sq. m)	= 1 are
640 acres	= 1 square mile	100 square ares (a)	= 1 hectare
1 square mile (sq. mi.)	= 1 section (of land)	100 hectares (ha)	= 1 square kilometre (sq. km)
36 sections	= 1 township		

WEIGHT

16 drams (dr.)	= 1 ounce	10 milligrams (mg) =
16 ounces (oz.)	= 1 pound	10 centigrams (cg) =
2000 pounds (lb.)	= 1 ton (tn.)	10 decigrams (dg) =
		10 grams (g) =
		10 dekagrams (dkg) =
		10 hectograms (hg) =
		1000 kilograms (kg) =

VOLUME

1728 cubic inches (cu. in.)	= 1 cubic foot	1000 cubic millimetres
27 cubic feet (cu. ft.)	= 1 cubic yard (cu. yd.)	1000 cubic centimetre
		1000 cubic decimetres

CAPACITY

Liquid		Liquid and Dry	
2 cups (c.)	= 1 pint	10 millilitres (ml)	=
2 pints (pt.)	= 1 quart	10 centilitres (cl)	=
4 quarts (qt.)	= 1 gallon (gal.)	10 decilitres (dl)	=
		10 litres (l)	=
Dry		10 dekalitres (dkl)	=
2 pints (pt.)	= 1 quart	10 hectolitres (hl)	=
8 quarts (qt.)	= 1 peck		
4 pecks (pk.)	= 1 bushel (bu.)		

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